Financial Frictions and the Market for Firms*

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May 2024

Abstract

In the U.S., one in four entrepreneurs purchased their business, with younger, smaller, and higher average revenue product of capital (ARPK) firms having the highest trading rates. After trade, capital outpaces output growth, reducing firms' ARPK over time. To explain these findings, we propose a model of entrepreneurship with a frictional market for firms where the trade of firms alleviates financial constraints. We show that the predictions from our theory are consistent with the cross-sectional and longitudinal facts. Our quantitative results indicate that firms' trade significantly improves allocative efficiency, with potential larger gains in less financially developed economies.

Keywords: misallocation, financial frictions, trade of firms, firm dynamics.

JEL classifications: E44, O47, G30.

* We are highly grateful to Virgiliu Midrigan and Diego Perez for their advice and support in the early stages of this project. We also thank Yan Bai, Santiago Bazdresch, Andrés Blanco, Corina Boar, Jarda Borovička, Lorenzo Caliendo, Gaston Chaumont, Juan Dubra, Sebastián Fanelli, Min Fang (discussant), Mark Gertler, Julian Kozlowski, Ricardo Lagos, Zachary Mahone, Timothy Munday (discussant), Sean Myers, Alessandra Peter, Luigi Paciello, Tommaso Porzio, Pablo Otonello, Yongseok Shin, Mathieu Taschereau-Dumouchel, and seminar participants at BSE SF 2023, CEBRA 2023, Columbia, Cornell, EEA-ESEM 2023, EWMES 2023, Hitotsubashi, HKU, IIES Stockholm, ITAM, NYU, Penn 2020 YES, Queens, Rochester, SEA 2020, SED 2023, St. Louis Fed, U de Montevideo, U of Toronto 2023 MacroDev Workshop, and WashU 2019 EGSC for valuable comments. Previously circulated as: "Entrepreneurship, Financial Frictions, and the Market for Firms". First Draft: May 2019.

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1 Introduction

Markets are the predominant allocation mechanism of modern economies. While markets that allocate resources across firms have been well-studied, such as capital and labor markets, less is known about the market where firms themselves can be bought and sold, i.e., *the market for firms*. In this paper, we argue that the role of this market is particularly relevant in economies where financial constraints are a pervasive feature of entrepreneurial activity. In such environments, who owns the firms matters for allocations, as credit-constrained entrepreneurs operate at a suboptimal scale, resulting in capital misallocation and lower aggregate output. The market for firms allows financially constrained entrepreneurs to sell their firms to other parties with more financial resources, potentially improving allocative efficiency in the economy.

We study the aggregate implications of the market for firms in multiple steps. Using cross-sectional microdata from U.S. business owners, households, and firms, we document that one out of four entrepreneurs purchased their business, with younger, smaller, and higher average revenue product of capital (ARPK) firms being more likely to be traded. To study post-trade firm dynamics, we use longitudinal firm-level data from several high-income countries to document that firms' output and capital grow significantly after trade, with capital outpacing output, reducing firms' ARPK. We also document that firms' average revenue product of labor (ARPL) remains constant after trade, while profitability and leverage fall. To explain these findings, we develop and quantify a macroeconomic model where agents can buy and sell firms in a frictional market. Gains from trading firms arise from financial frictions, namely credit constraints and incomplete financial markets, and preference shocks that capture alternative motives to trade firms. We show that the cross-sectional and longitudinal facts are consistent with the main prediction of our theory that the trade of firms alleviates financial constraints. Lastly, we use our quantitative framework as a laboratory to study the relevance of this market for the macroeconomy. Our results indicate that firms' trade improves allocative efficiency with more significant potential gains in less financially developed economies.

We start our empirical analysis by using multiple data sources to establish salient cross-sectional features of the market for firms in the U.S. economy. First, we document that one out of four entrepreneurs (around 23% to 26%) in the U.S. acquired their business by purchasing an existing firm, implying an annual trade rate of 3%. Compared to other assets, private firms are traded less frequently than real estate but more so than patents.[1]

[1] Berger and Vavra (2015) reports that 5% of houses are traded annually, higher than the 3% trade rate for private firms. Akcigit, Celik, and Greenwood (2016) documents that 16% of U.S. patents have been traded, smaller than the 26% we find for private businesses.

Second, we document two salient characteristics of business buyers. Our first finding reveals that 66% of buyers have never been entrepreneurs before purchasing their current firm. This finding suggests that buying an existing firm is a relevant way to enter entrepreneurship. Besides capturing the trading frequency of private firms, our theoretical framework incorporates this novel feature about households possible transitions into entrepreneurship through the market for firms. Our second finding shows that the average wealth of firm buyers is about three times that of the average household. This evidence will serve us to test our theory of financial frictions being a relevant driver for firms' trade.

Third, we establish novel cross-sectional facts about the trading frequency and firms' observable characteristics. We document that younger, smaller, and higher ARPK firms have the highest trading rates. These cross-sectional results about firms' characteristics and trade frequency are informative about the underlying motives behind firms' trade. Both firms' age and size are associated with financial constraints (Hennessy and Whited, 2007; Hadlock and Pierce, 2010). Furthermore, firms' ARPK is informative about their access to external finance, as credit-constrained firms may have high returns but cannot increase their investment. By introducing financial frictions as a micro foundation that generates gains from trading firms, our model can account for these cross-sectional facts about the firms more likely to be bought and sold in the market for firms.

In addition to our cross-sectional analysis, we document novel post-trade firm dynamics using longitudinal firm-level data to shed light on the different motives driving firms' trade. Due to data limitations, we use data from several high-income European countries, which are the most comparable to the U.S. economy. We document that, on average, firms' capital increases by 51%, while output increases by 25% five years after trade. As a result of the joint dynamics of capital and output, ARPK sharply falls. In contrast, the change in firms' ARPL is minimal and is not statistically different from zero. We also document that profitability, measured by the profits-to-capital ratio, decreases. Finally, we document that firms' leverage falls after trade despite the significant capital increase. The dynamics of firms' capital and debt show that businesses traded in the market for firms receive significant equity injections from their new owners. As we explain below, the post-trade firm dynamics in the data are consistent with the trade of firms alleviating financial constraints, as predicted by our model, and put a high bar for alternative theories on firms' trade that would need to account for these longitudinal facts.

Motivated by these findings, we develop a heterogeneous agent model of entrepreneurship and frictional trade of firms. Our model economy is populated by a continuum of households, which can be firm owners or workers. Firm owners can trade or shut down their firms, while workers can become business owners by buying an existing firm or through a startup shock. There are credit constraints and incomplete financial markets, so households are subject to uninsurable idiosyncratic risk. On the one hand, firm owners face the risk associated with the quality of their firm, which evolves stochastically. On the other hand, workers are subject to shocks to their labor efficiency.

We characterize firms through the quality of an entrepreneurial project, which is indivisible, rival, and excludable. Firms' quality captures all features characterizing a firm outside of labor and capital inputs, such as organizational capital or intangible assets. Firms' owners can produce the final consumption good with a technology that combines capital, labor, and the firm's quality. Besides the firms owned by a single household, which we call private firms, there is a second production sector with a representative public firm. Both sectors produce the same good, which can be used for consumption or savings in a risk-free asset. There is also a financial intermediary that, each period, takes the savings from the households and rents capital to the firms.

Our empirical results show that private firms are traded infrequently, which motivates using a search-theoretic approach to model this market. Specifically, we model the market for firms through a decentralized market subject to search frictions and bilateral random matching. A Nash bargaining protocol between sellers and buyers determines the trading price. One interpretation of these assumptions is that agents can valuate only one firm at a time, which delays trade. This setup is suitable for our quantitative analysis and allows the model to match relevant features about the market for firms.

Exchanges in the market for firms are voluntary. Hence, a necessary condition for trade is that agents have different valuations for the same firm, with the buyer having a higher valuation than the seller. Heterogeneity in firms' valuations arises from three sources in our theory: credit constraints, incomplete markets, and preference shocks. Credit constraints and incomplete markets generate an endogenous motive to trade. For a given firm, unconstrained agents attain higher profits, grow the firm faster, and bear the risk better than constrained agents. By transferring firms between agents with different wealth levels, the market for firms can improve allocative efficiency. In addition, we assume that potential sellers are subject to idiosyncratic preference shocks that parsimoniously capture other motives to trade firms that we do not explicitly incorporate in our theory.

We calibrate the model to match several features of the U.S. economy. Namely, we target moments related to the role of entrepreneurs, the income and wealth distribution across households, the relative importance of the private business sector, and key four moments about the market for firms documented in the empirical part of the paper.

After quantifying the model, we perform three exercises evaluating testable predictions of our theory about financial frictions being a relevant motive to trade firms. The first two predictions are related to the cross-sectional evidence about the market for firms. First, we test the prediction of our model regarding business buyers' characteristics and find that, as in the data, buyers are up to three times wealthier than the average household. Second, we compare the model-simulated relations between trade rates and firms' observable characteristics with their empirical counterparts. Consistent with the data, our model predicts that younger, smaller, and higher ARPK firms have the highest trading rates. The fact that these groups of firms are associated with binding credit constraints in our model, which generates gains from trading firms, explains this result.

In our third exercise, we test the implications of our theory for post-trade dynamics. Our theory predicts that firms' trade alleviates financial constraints. Hence, before trade, firms operate with lower capital and higher ARPK relative to their unconstrained level. After trade, capital in our model grows more than output, reducing firms' ARPK over time, as in the data. We show that although the dynamics in our model are faster, with firms immediately jumping closer to their optimal scale, the overall effect five years after trade in our model aligns exceptionally well with the data. Financial frictions in our model do not distort labor input decisions, which implies that firms' ARPL is constant, which is in line with the flat pattern in the data. We also show that credit-constrained firms in our model have a distorted profit-to-capital ratio higher than their unconstrained level. Thus, like the ARPK dynamics, capital grows more than profits, reducing firms' profitability after trade, as in the data. Finally, as firms' buyers tend to be wealthier than sellers in our model, most of the additional capital comes from owners' equity, resulting in a decrease in firms' leverage comparable in magnitude to that in the data. Overall, our model is consistent with a wide range of cross-sectional and longitudinal moments that we did not target in our calibration exercise. Instead, these patterns arise from the key endogenous motive to trade firms we study in this paper due to financial frictions in the economy.

After providing evidence consistent with testable predictions from our theory, we quantify the macroeconomic implications of the market for firms as a mechanism that allocates productive projects and available resources in the economy. We perform two counterfactual experiments. In our first experiment, we take our baseline model and analyze a scenario in which the market for firms shuts down. Closing this market implies a fall in aggregate entrepreneurial output and total factor productivity (TFP) of 9.1% and 2.2%, respectively. This result is explained by a lower entrance into entrepreneurship and a poorer capital allocation when this market is absent.

The previous exercise shows that the market for firms alleviates the capital misallocation caused by financial frictions. To better understand the TFP gains from this market, in our second experiment, we quantify the additional external financing that entrepreneurs in an alternative economy without the market for firms would require to achieve the same allocative efficiency as in our baseline economy with firms' trade. We find that the no market economy requires a relaxation of financial constraints such that the aggregate debt-to-capital ratio of private firms rises by 14 percentage points (p.p.), from 0.35 to 0.49, which is a significant increase. To put this number in perspective, private firms' leverage in the U.S. dropped by 5 p.p. during the Great Recession.

Finally, we investigate the interaction between financial development and the trade of firms. In our model, the functioning of both markets for credit and firms determines the allocation of productive resources. As in the finance and misallocation literature, our model implies that higher levels of financial development lead to a better allocation of capital and higher TFP. Unlike previous work, we show that aggregate TFP can increase through a better-functioning market for firms for any level of financial development. The market for firms can play an even more important role in economies with tighter credit frictions, as the potential gains from trading firms are higher. Consistent with this prediction, we document that post-trade firm dynamics are more pronounced in middle-income and less financially developed countries, with capital and output increasing twofold than in high-income economies. Overall, our results indicate that the market for firms can be a relevant substitute for debt financing in economies with less developed credit markets.

Related Literature Our paper contributes to the following strands of the literature.

Trade of Firms and the (Re)Allocation of Productive Resources. Our work primarily contributes to the literature that studies the trade of private firms as an allocation mechanism. Earlier work includes Holmes and Schmitz (1995), which studies an economy where owners have firm-specific match qualities, and Caselli and Gennaioli (2013), which analyzes the trade of family firms when heirs have low managerial talent. Closest to our work are two contemporaneously developed papers studying firms' trade in heterogeneous agents models with entrepreneurship. Gaillard and Kankanamge (2020) study a model where mature firms can be traded or liquidated upon forced exit shocks. In our empirical analysis, however, we show that younger firms, which are not allowed to be traded in that paper, have the highest trading rates. Mahone (2023) analyzes an economy where firms' trade is solely driven by exogenous preference shocks, which we find to explain less than one-third of the transactions in our quantitative model. Unlike these papers, we study an economy where gains from trade arise *endogenously* from credit constraints and incomplete financial markets. In our setup, the market for firms is multidimensional, with buyers' and sellers' wealth, in addition to firms' quality, playing a first-order role in determining trading surpluses and equilibrium prices. Our paper is the first in this literature to use longitudinal data to document post-trade firm dynamics, which are highly informative about the empirical relevance of different theories on firms' trade. We show that our novel cross-sectional and longitudinal facts are consistent with the main prediction of our theory that the trade of firms alleviates financial constraints.

Entrepreneurship in Macroeconomics. Our theoretical framework builds on the literature on heterogeneous agents models with entrepreneurship that started with the seminal work of Quadrini (2000) and Cagetti and De Nardi (2006). We contribute to this literature by extending the framework to allow for the trade of entrepreneurial projects in a frictional market for firms. Our model can account for the fact that one out of every four entrepreneurs in the U.S. purchased their business, and over two-thirds were not entrepreneurs before acquiring their firm.

Finance and Misallocation. Our paper also contributes to the literature on financial frictions and capital misallocation as a source behind TFP differences across countries (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014; David and Venkateswaran, 2019). We show that the market for firms can reduce the capital misallocation caused by financial frictions, especially in less financially developed economies where the gains from trading firms are higher. We provide evidence consistent with this prediction of our model by documenting that post-trade firm dynamics are twice as large in middle-income and less financially developed countries than in high-income countries.

M&A in Finance and Macroeconomics. Our paper is also related to the literature in finance and macroeconomics studying mergers and acquisitions (M&A), where acquirers are existing firms. This is a crucial distinction relative to our paper, where households can buy and sell firms. Indeed, most buyers in our analysis have never been entrepreneurs before purchasing their firm. Significant contributions to this literature include Jovanovic and Rousseau (2002) and Rhodes-Kropf and Robinson (2008), which study theories of high-buys-low and like-buys-like, respectively. More recent work studies M&A in firm dynamics models à la Hopenhayn (1992). For example, David (2021) quantifies the implications of firms' mergers in an environment with complementarities, while Bhandari, Martellini, and McGrattan (2022) studies a model of acquisitions where intangible capital is subject to convex adjustment costs. Our empirical analysis also relates to the literature in corporate finance that studies the financial rationale behind M&A. For example, Liao (2014) and Erel, Jang, and Weisbach (2015) analyze post-M&A firm dynamics and find evidence that acquisitions relieve financial constraints in target firms. We contribute to this literature by providing novel evidence for a broader sample of transactions, including younger and smaller firms and involving households (e.g., workers buying firms in our model), usually not covered in the M&A data. We document that post-trade firm dynamics are consistent with the trade of firms alleviating financial constraints. Our results indicate that financial frictions play an important role in firms' trade beyond M&A deals.

Aggregate Implications of the Market for Ideas. Finally, from a theoretical perspective, our paper relates to the literature studying the implications of the trade of ideas (Silveira and Wright, 2010; Akcigit, Celik, and Greenwood, 2016). As in that literature, we use a framework characterized by bilateral meetings subject to search frictions, where the likelihood of trade depends on meeting probability parameters and the endogenous distribution of agents in the economy. Different from papers of non-rival ideas (Lucas and Moll, 2014; Perla and Tonetti, 2014), diffused through imitation, firms are rival and excludable in our environment, and hence sellers need to be compensated by buyers. Under credit constraints and incomplete financial markets, the aggregate implications of the market for firms are determined by how this market affects allocative efficiency in the economy.

Outline The rest of the paper is organized as follows: Section 2 presents our empirical analysis; Section 3 presents the model; Section 4 describes our parameterization; Section 5 describes the main properties of our model; Section 6 evaluates several testable predictions of our theory on the trade of firms and discusses alternative theories; Section 7 presents our aggregate results; and finally, Section 8 concludes.

2 Empirical Analysis

In this section, we use microdata from business owners, households, and firms to document relevant cross-sectional and longitudinal facts about the market for firms. In the cross-section, we document the share of entrepreneurs who purchased their businesses, the previous occupations and wealth of business buyers, and the characteristics of the firms traded most frequently. Our longitudinal analysis documents novel facts about the joint dynamics of firms' capital and output after trade. We also document additional results related to the post-trade dynamics of firms' profitability, labor inputs, and leverage.

2.1 Cross-Sectional Analysis

This section presents our cross-sectional analysis of the U.S. market for firms. We first describe our data sources and then present our cross-sectional facts. Appendix A provides robustness checks and additional exercises related to our cross-sectional analysis.

2.1.1 Data Sources

We use three different surveys related to private firms, their characteristics, and the characteristics of their owners.[2] First, our main data source is the Survey of Business Owners (SBO) Public Use Microdata Sample (PUMS). This survey provides comprehensive information about businesses and business owners. In particular, about how they acquired their business. The PUMS sample is representative of all non-farm private businesses in the U.S. and is available for the year 2007.

Second, we use nine waves of the Survey of Consumer Finances (SCF) covering the period between 1989 and 2016. Importantly, the SCF includes detailed information about households' income and balance sheets, which we will use to discipline our quantitative model's income and wealth distribution. Additionally, this survey asks business owners how they acquired their firms. The information in the different waves of the SCF allows us to study how the ownership of firms has evolved over time.

Finally, we use data from the Kauffman Firm Survey (KFS). The KFS is an eight-year panel of firms that started operations in 2004 and were followed through 2011. Unlike the previous datasets, the KFS contains information about firms' balance sheets, allowing us to compute firm-level capital. However, the KFS sample is not representative of the entire private sector, as it is a survey of startups. We will account for this fact when comparing the KFS evidence with data simulated from our model. Appendix A.1 presents further details about these datasets, variables' definitions, and our sample selection criteria.

Entrepreneurs Our cross-sectional empirical analysis focuses on *entrepreneurs* as the observation unit. We follow Cagetti and De Nardi (2006) and define entrepreneurs as self-employed individuals who own a business and have an active management role in it. Given our interest in the trade of firms, we restrict to the entrepreneurs with at least one employee.[3] According to the 2007 SCF, entrepreneurs represent 6% of households. As previous studies have documented, although entrepreneurs represent a small fraction of the population, they earn 20% of total income and hold 33% of total wealth. In our calibration strategy, we will target these key features of the role of entrepreneurs in the economy.

Throughout our analysis, we assume that each entrepreneur owns and manages only one firm. This assumption implies that the number of firms traded every period equals the number of entrepreneurs that trade their firms. Hence, we use both terms interchangeably. Our assumption relies on the fact that, in the SCF, more than 80% of entrepreneurs own

[2] In Appendix A, we provide additional evidence using the Annual Survey of Entrepreneurs (ASE). Our main findings are consistent with the results obtained from that complementary data source.

[3] We focus on entrepreneurs with a positive number of employees to exclude the cases of self-employed individuals whose businesses might not be transferable. In Appendix A.2, we present results considering all entrepreneurs (with employer and non-employer firms).

only one firm (see Table A.11). Furthermore, according to the SBO, more than 74% of the private firms in the economy have only one entrepreneur and more than 96% of the firms have at most two entrepreneurs (see Table A.12).[4]

2.1.2 How do Entrepreneurs Acquire Their Firms?

Share of Traded Firms We start our analysis using the SBO and the 2007 SCF to document how entrepreneurs acquire their firms.[5] We focus on three main types of acquisitions: founding a firm, purchasing an existing firm, and inheriting or other kinds of acquisition. Table 1 shows that two-thirds of entrepreneurs acquire their firms by founding their businesses. Also, it shows that 9% to 12% of entrepreneurs acquired it through inheritance or other types of acquisition. The most relevant number for our analysis is that 23% to 26% of the entrepreneurs in the U.S., depending on the survey, acquire their business by purchasing an existing firm.[6]

Table 1: Share of Entrepreneurs by Business Acquisition

	Founded	Purchased	Inherited/Other
SBO	65.2%	25.5%	9.3%
SCF	65.3%	22.7%	12.0%

Source: SBO and SCF for the year 2007.
Notes: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm, and (iv) the firm has at least one employee. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

In Appendix A.2.1, we verify the robustness of these findings. Table A.4 shows that our results are robust to several alternative definitions of entrepreneurs, such as focusing on firms' owners with majority equity shares. Table A.5 shows that the results are almost identical when we compute the share of traded firms at the firm-, instead of the entrepreneur-, level. Table A.6 shows that the presence of franchises does not drive our results. Finally, using different waves of the SCF, we compute the share of traded firms over time. Figure A.2 shows that the share of entrepreneurs that purchased their business has declined in the last three decades but has been fairly stable since 2007.

Firms' Trade Rate The previous results refer to the *stock* of firms that have been traded at any point in the past. We are also interested in the annual frequency of trade, i.e., the trade *rate*. We estimate the percentage of firms traded every year using two

[4] In this line, Appendix A.3.3 documents that private firms' ownership and management are highly concentrated (Figure A.3), even for the economy's oldest and largest privately held firms (Figure A.4).

[5] Specifically, the SBO asks: "*How did [the owner] initially acquire ownership of this business?*". Similarly, the SCF asks business owners: "*How did you first acquire this business?.*"

[6] Using the same data sources, Gaillard and Kankanamge (2020) and Mahone (2023) contemporaneously documented similar numbers for the stock and frequency of firms' trade.

strategies. The first strategy looks at the percentage of firms purchased in the SBO and SCF data in the same year of the survey. The second strategy, as Appendix A.4 describes, uses the law of motion of the stock of traded firms as a function of firms' entry, exit, and trade rates. Either strategy implies that around 3% of the firms are traded every year.

2.1.3 Buyers' Characteristics

Buyers' Previous Occupation Using the SBO, we can obtain information regarding entrepreneurs' previous occupations. We found that 66% of the entrepreneurs who purchased their firm have never been self-employed. Hence, most likely, these individuals were in the labor market before acquiring their businesses. This result indicates that buying an existing firm is a relevant channel for entering into entrepreneurship, which, to the best of our knowledge, has not been studied before. Besides capturing the trading frequency of private firms, our theoretical framework will incorporate this novel feature about households' possible transitions into entrepreneurship through the market for firms.[7] Table A.7, in the appendix, shows that this result is robust to alternative samples and definitions, and Table A.8 shows that this number is similar across firms' age and size distributions.

Buyers' Wealth Using the SCF, we can identify the entrepreneurs that recently purchased their businesses and measure their wealth. Table C.3, in the Appendix, reports the wealth of the average buyer relative to the wealth of the average household and the wealth of the average entrepreneur. We consider two definitions of wealth, with and without business wealth. The average buyer is 2.7 times wealthier than the average household and 0.8 times relative to the average entrepreneur, excluding business wealth. Considering total wealth, these numbers are 3.8 and 0.7, respectively. Thus, business buyers are considerably wealthier than the average household but less wealthy than other entrepreneurs. In Section 6, we compare this evidence with the characteristics of buyers in our model and discuss how the fact that business buyers tend to be wealthy is consistent with our theory of financial frictions being an important driver behind the trade of firms.

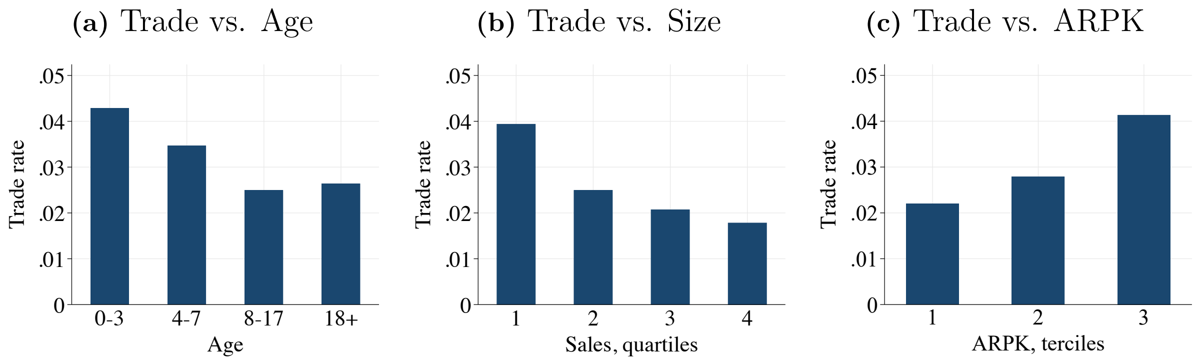
2.1.4 Trade Rate and Firms' Characteristics

We now document novel cross-sectional evidence for trade frequency conditional on firms' observable characteristics. We focus on three attributes: firms' age, size, and the average revenue product of capital (ARPK).

Firms' Age We document the trading frequency by firm age using data from the SBO. For this analysis, we focus on the firms sold in or after 2007, the same year as the survey. Thus, we measure firms' age as the difference between the survey year and the year the

[7] We consider the question in the SBO: "*Prior to acquiring this business, had the owner ever owned a business or been self-employed?*" This number should be interpreted as a lower bound of our non-entrepreneur definition (i.e., the complement of being an entrepreneur).

Figure 1: Trade Rate by Firms' Characteristics



Source: SBO and KFS.
Notes: Panels (a) and (b) use data from the 2007 SBO, and panel (c) uses data from the KFS. In panels (a) and (b) trade is computed using information from firms sold in or after 2007. Size is measured using firms' sales. Panel (c) uses data from KFS. Trade is computed using the firms sold during the years of the sample. We compute this every year and then take the average across time. Average revenue product of capital (ARPK) is measured by sales over capital of the previous year to the sale. Trade rates are normalized to match the aggregate of our baseline calculations.

firm was founded.[8] Panel (a) of Figure 1 presents the trade rate across different age bins. The figure shows that the youngest firms (0-3 years old) have the highest trading rates, with a trading frequency around 50% larger than those in the older age groups. After age eight, the relationship is relatively flat among the oldest firms.

Firms' Size We also employ the SBO to study the relation between trade and firm size, focusing on the firms sold to measure their size before the exchange occurs. For this, we look at the sample of business owners that sold their firm in or after 2007 and measure size using data from the previous year of operation. Thus, we relate the trade probability at t against the firm's size at $t-1$. We measure firm size using sales. Panel (b) of Figure 1 presents the probability of trade for different quintiles of the size distribution. We find that the frequency of trade and firm size are negatively related. Thus, the smaller firms, measured by sales' bottom quartile, have the highest trading probabilities.[9] While smaller firms have the highest trading frequencies, Table A.9 shows that the largest firms in the SBO are more likely to have been traded in the past. This evidence suggests that firms tend to be small when traded but grow significantly afterward. These cross-sectional results are consistent with the post-trade firm dynamics we document in Section 2.2.

Firms' ARPK Finally, we document the relation between the trade rate and firms' ARPK. We measure ARPK using data from the KFS, which includes information about

[8] Alternatively, we could measure traded firms' age using information from the buyer side by looking at firms purchased in 2007. However, recently acquired firms incorrectly reported as newly established might contaminate this measure of firms' age, especially for trades at age 0. This issue is unlikely to occur in the case of recently sold firms, as sellers are the ones reporting firms' year of foundation.

[9] Figure A.1 shows that the results are very similar when we measure size using firms' total payroll.

firms' balance sheets that allow us to compute a firm-level measure of capital. As the analysis for size, we relate firms' ARPK at period $t-1$ against the probability of trade at t, which we measure as the share of owners that report having sold or merged their business. Panel (c) of Figure 1 shows a positive relation between the frequency of trade and ARPK, with the top terciles ARPK firms having the highest trading rates.

In sum, we document that younger, smaller, and higher ARPK firms have the highest trading rates. These results regarding firms' observable characteristics and trade frequency provide insight into the underlying mechanisms behind firms' trade. As we show below, these cross-sectional facts are consistent with the prediction of our theory that financially constrained firms are more likely to be traded.

2.2 Post-Trade Firm Dynamics

This section uses longitudinal data from several high-income countries and an event analysis framework to document novel facts about firm dynamics after trade. First, we describe the data and discuss how we identify trades in the market for firms. We then present our empirical specification and describe our results.

2.2.1 The Orbis Database

Due to data limitations for the U.S., for which we only have cross-sectional data, we use Orbis Historical, an extensive firm-level panel database covering millions of companies worldwide, to provide novel empirical evidence about post-trade firm dynamics. We focus on a sample of European private firms, which are the ones with the best coverage in Orbis. The historical product of Orbis covers 71% of the gross national output of the countries in our sample and captures well the firm-size distribution documented in official sources (Kalemli-Özcan et al., 2023). For our baseline results, we consider eleven high-income European countries that are the most comparable to the U.S. economy.[10] Appendix B.1 provides a detailed description of this data, discusses our sample selection, presents descriptive statistics, and reports the variables' definitions. Orbis contains income and balance sheet statements from 1996 to 2019, from which we compute firm-level measures of capital, labor, and output. From 2007 onward, the data reports annual ownership records with the name and shares of firms' equity holders.

Identifying Trades in the Market for Firms We use the ownership files in Orbis to identify transactions in the market for firms. In detail, we define the trade episodes as the years in which we observe a change in the majority owner of a firm (equity share above 50%). Therefore, we identify firms' trades by tracking majority owners' identities over

[10]The eleven high-income countries included in our baseline analysis are: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. Appendix B.3 presents additional results for middle-income European countries.

time. As Appendix B.2 describes, we identify changes in owners' names using a string similarity algorithm that excludes changes in names that are spurious or that are likely related to inheritances or family-related transfers.[11]

2.2.2 Empirical Specification

After identifying trade episodes, we run a non-parametric regression to analyze post-trade firm dynamics. Let i denote a firm and t time. We normalize the trading year to $t = 0$. Then, for each variable of interest, x_{it}, we estimate

$$\log x_{it} = \beta_0 + \sum_{h \in \mathcal{T}} \beta_h \mathrm{D}_{it}^h + \gamma \mathbf{c}_{it} + \epsilon_{it}, \tag{1}$$

where $\mathcal{T} = \{-1, 1, \ldots, 5\}$ and D_{it}^h is a indicator variable equal to 1 if time t corresponds to the period h around the trading episode. Thus, as $\mathcal{T}$ indicates, we study firm dynamics from one year before up to five years after trade. For our analysis, we restrict to firms we observe for at least five consecutive years from $t = -1$ to $t = 3$. The vector $\mathbf{c}_{it}$ denotes our control variables, including country and NACE 4-digit sector fixed effects. Given the relevance of firm age for firm dynamics (Haltiwanger, Jarmin, and Miranda, 2013; Kochen, 2023), we also control for firms' age at $t = 0$, the year the firm was traded.

2.2.3 Firm Dynamics After Trade

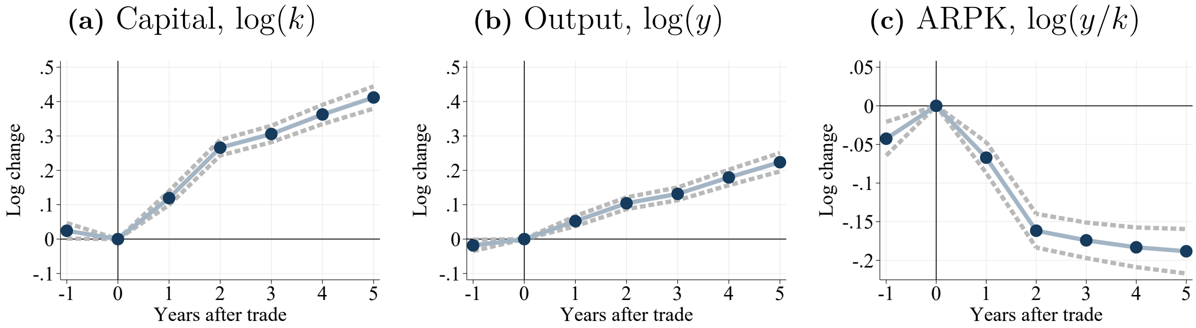
Figure 2 presents the results from estimating (1) for firms' capital, output, and ARPK. Panel (a) shows that, on average, firms' capital significantly grows by 0.31 log points (36%) three years after trade and by 0.41 log points (51%) after five years. These results indicate that traded firms' five-year capital growth is twice as large as the average firm in our sample, with capital growing at 0.05 log points (5%) per year.[12] Panel (b) shows that output increases, but to a lower extent, in 0.13 and 0.22 log points (14% and 25%, respectively) after three and five years, respectively. As a result of the joint capital and output post-trade dynamics, ARPK sharply falls by -0.17 and -0.19 log points (-16% and -17%, respectively) three and five years after trade. The appendix presents additional robustness exercises supporting our key novel findings about post-trade firm dynamics. In Figure B.1, we extend the episode window backward and show that there is no clear pre-trade trend as the estimated $\hat{\beta}_h$ for $h < 0$ is close to zero before firms are traded, for both capital, output, and ARPK. Figure B.2 shows that our results are robust to alternative specifications, such as including firm-level fixed effects as controls.

In addition to our main result about the joint dynamics of capital and output, Figure 3 presents the post-trade firm dynamics of the average revenue product of labor (ARPL),

[11] For example, our algorithm excludes changes in owners' names, such as "Federico Kochen" to "Kochen Federico" or "Luis Guntin" to "Rafael Guntin", as the last latter is likely related to inheritances.

[12] See Table B.1 in the appendix for our sample descriptive statistics.

Figure 2: Capital, Output, and ARPK Dynamics After Trade



Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

profitability, and leverage. As we explain in Section 6.4 below, the evolution of these ratios is informative about the empirical relevance of different motives behind firms' trade. Panel (a) shows the results for firms' ARPL, defined as output over labor costs, $\log(y/wl)$. Differently from the ARPK, the change in firms' ARPL is minimal and is not statistically different from zero in any of the years after trade. As Figure B.3 in the appendix shows, the flat profile of ARPL is because firms' labor costs grow at the same rate as output after firms' trade.[13] Regarding firms' profits, panel (b) shows that profitability, measured by the profits to capital ratio $\log(\pi/k)$, falls by 0.17 log points (16%) five years after trade. Figure B.4 shows that this result holds despite firms' profits growing after trade. That figure also shows that the results are similar if we measure profitability using firms' return on assets (ROA).[14] Finally, regarding firms' financing, panel (c) presents the dynamics of leverage, $\log(b/k)$. This panel shows that leverage falls by 0.08 and 0.11 log points (8% and 10%) and three and five after trade, respectively. Figure B.5 in the appendix shows that while debt, on average, rises after trade, capital grows significantly more. The joint dynamics of capital and debt, captured by firms' leverage, show that businesses traded in the market for firms receive significant equity injections from their new owners.[15]

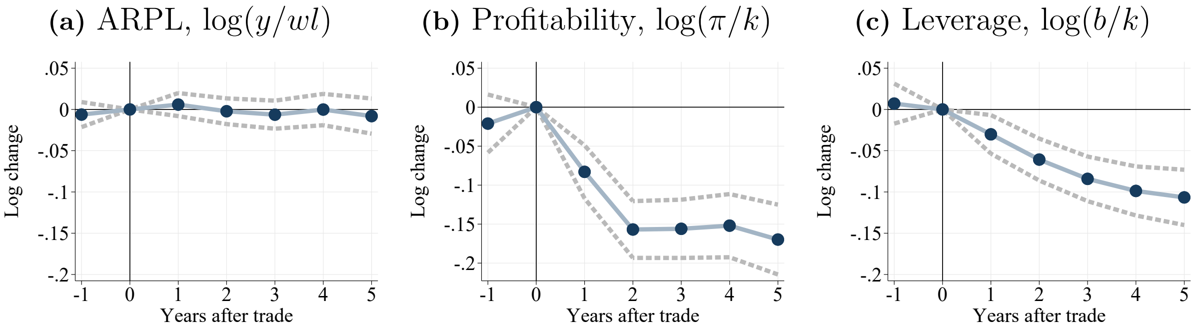
High- and Middle-Income Countries So far, our longitudinal analysis has focused on a group of high-income countries, arguably the most similar to the U.S. economy. In

[13] Our findings are consistent with Bau and Matray (2023), which studies firm dynamics after a financial liberalization episode in India. Table 4 of that paper shows that, after the liberalization that reduced financial constraints, firms' ARPK sharply fell by 20%, while ARPL slightly decreased by less than 5%.

[14] If firms' profits π are negative, they will not be included in our log changes measures for profitability and ROA. Panel (b) of Figure B.4 shows that the share of firms with non-positive profits, $\pi \leq 0$, slightly falls, but does not substantially change, after firms' trade.

[15] Our baseline results are for net financial leverage (Welch, 2011), defining b as financial debt minus cash, which can take negative values. Panel (b) Figure B.5 shows that the share of firms with non-positive net financial debt, $b \leq 0$, slightly rises, consistent with firms paying off their debt after being traded. Panel (d) of that figure shows that the decline in leverage after firms' trade is larger when only using financial debt in the numerator, which shows the effect of excluding negative values when taking logs.

Figure 3: ARPL, Profitability, and Leverage Dynamics After Trade



Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Appendix B.3, we present additional results for the high- and middle-income European countries in Kochen (2023). Figure B.6 in the appendix shows that post-trade firm dynamics are considerably amplified in middle-income and less financially developed countries, as both capital and output increase more than in high-income countries. More importantly, the decline in firms' ARPK in middle-income countries, five years after trade, is more than twice as large as the one documented in the high-income region. As we describe below, this cross-country evidence is consistent with our theory that predicts the trade of firms playing an even more relevant role in economies with less developed financial markets.

To summarize, we documented several novel facts about post-trade firm dynamics. Our longitudinal results show that, five years after trade, output and capital increase substantially, with capital outpacing output, leading to a sharp decline in firms' ARPK. Furthermore, unlike ARPK dynamics, the post-trade change in firms' ARPL is not statistically different from zero. We also documented that firms' profitability decreases while profits rise after trade. Finally, regarding firms' financing, we find that leverage falls after trade despite the significant capital increase, reflecting that new owners do considerable equity injections to recently purchased firms. As we show below, by introducing financial frictions as a micro foundation that endogenously generates gains from trading firms, the model we now describe can account for *all* the cross-sectional and longitudinal facts of the market for firms previously documented.

3 A Model of Entrepreneurship and Trade of Firms

In this section we develop a general equilibrium heterogeneous agent model with four key elements: endogenous occupational choice between entrepreneurship and labor, uninsurable income risk for workers and entrepreneurs, firm-level credit frictions, and a frictional market in which firms can be bought and sold.

3.1 Environment

Our model economy is inhabited by a continuum of households in $[0, 1]$. Households can have two possible occupations: *firm owners* or *workers*. Firm owners can buy and sell firms and choose whether to operate their current firm and be *entrepreneurs* or close the firm and become workers. Workers can become firm owners by acquiring a firm or through some exogenous *startup* shock. We explain the transitions between these two occupations in further detail below.

Besides the firms owned and managed by individual households, which we call *private firms*, there is a second sector of production that features a representative *public firm*. Both sectors produce the same good, which can be used for consumption or savings. Capital is produced by a *financial intermediary* which, each period, takes savings from households and rents capital to the firms. All households own the public firm and the financial intermediary in equal shares.[16]

Time is discrete and infinite, and each time period is divided into two stages. The trade of firms occurs in the first stage, which we call the decentralized market, or *DM*. We assume that, in the market for firms, households meet bilaterally subject to *search frictions*, which may restrain the frequency and the type of the matches. All production, consumption, and saving decisions take place in the second stage, which we call the centralized market, or *CM*.

Households Households have preferences over consumption c represented by a constant relative risk aversion (CRRA) utility function

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}$$

where σ is the risk aversion coefficient.

They are heterogeneous in their occupation and their asset holdings a_{it}. Assets are subject to a non-borrowing constraint, $a_{it} \geq 0$, and are deposited with the financial intermediary, which pays a risk-free interest rate of r for the deposits. There is no aggregate uncertainty in this economy. However, households face idiosyncratic uninsurable risks.

Firm owners are endowed with a private firm that enables the owner to produce the final consumption good with a technology that uses capital, labor, and the firm's quality. We describe this technology below. The quality of the firm, denoted by z_{it}, is stochastic

[16] Alternatively, we could have assumed that the intermediary and the public firm issue equity shares, which are traded between households in a frictionless centralized market. This setup is analogous, as assets and shares holdings would be indeterminate. Below we assume that the intermediary and the public firm make zero profits. Thus, this modeling choice is not crucial for the analysis.

and evolves according to the law of motion

$$z_{it+1} = \begin{cases} z_{it} & \text{with pr. } \gamma \\ z' \sim \mathcal{P}(z_{min}, \eta_z) & \text{with pr. } (1-\gamma) \end{cases}$$

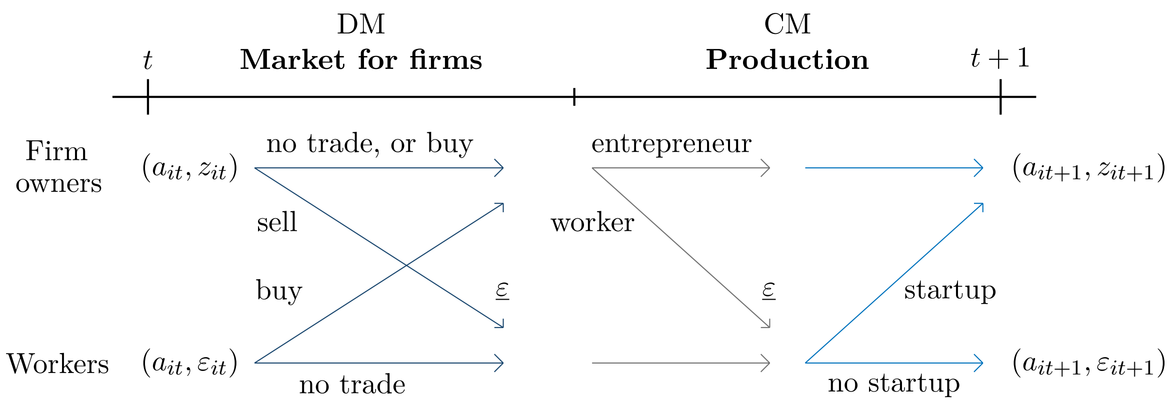
where $\mathcal{P}$ denotes a Pareto distribution with scale and a shape parameters z_{min} and η_z, respectively. The $(1-\gamma)$ shock can be interpreted as changes in market conditions that affect the profitability of entrepreneurial projects as in Buera, Kaboski, and Shin (2011).

On the other hand, workers are endowed with one unit of labor, which they supply inelastically, and are heterogeneous in their labor efficiency ε_{it}. We assume that the logarithm of the labor market efficiency evolves according to an AR(1) process with persistence ρ_ε and volatility σ_ε. Specifically,

$$\log \varepsilon_{it+1} = \rho_\varepsilon \log \varepsilon_{it} + \sigma_\varepsilon u_{it+1},$$

where u is a standard normal random variable.

Figure 4: Transitions Between Occupations



Regarding the transitions between occupations, workers can become firm owners by purchasing an existing firm or through an exogenous startup shock at the end of the period. At the beginning of the production stage, firm owners face an occupational choice. They decide whether to operate their firm or shut down the firm and become workers. Upon exit or selling, previous firm owners lose their firm's quality and enter the labor market with the lowest labor market efficiency $\underline{\varepsilon}$. We interpret this low entry value as potential costs associated with entrepreneurship, such as lack of experience in the labor market.[17] Figure 4 presents a graphical description of the transitions between occupations.

[17] We make this assumption for tractability, such that two state variables characterize business owners and workers. Without this assumption, and hence suppose that previous owners get a higher value $\tilde{\varepsilon}$, workers with $\varepsilon < \tilde{\varepsilon}$ would have an incentive to buy a low-quality firm and then immediately exit to improve their labor efficiency. Note that while the distribution of ε is bounded below by 0, in our numerical solution, we take $\underline{\varepsilon}$ to be the lowest value on the ε grid, which is a positive number.

In this setup, the budget constraint of an entrepreneur, defined as a firm owner that decides to operate, with states (a_{it}, z_{it}) is given by

$$c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

and the budget constraint of a worker with states $(a_{it}, \varepsilon_{it})$ is

$$c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1} + \Pi^p + \Pi^f,$$

where π are the profits of the entrepreneur's private firm, w is the labor market wage, Π^p and Π^f are the public firm's and the financial intermediary's profits, respectively.

Private Firms Private firms are endowed with a technology that uses capital k_{it}, labor l_{it}, and an *entrepreneurial project* z_{it} to produce the final good according to

$$y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu}$$

where $\theta + \nu < 1$. The decreasing returns to scale assumption implies that all private firms have an optimal operation scale as in Lucas (1978).

Private firms rent capital and hire workers every period. Hence, they are characterized only by the quality of z_{it}. Private firms are indivisible, rival, and excludable. These features are an important distinction between our model of the trade of firms and the literature on the trade of ideas.[18] Different values of firms' quality z_{it} capture all features characterizing a firm outside of labor and capital inputs. For example, firms' organizational capital or intangible assets.[19]

We assume that entrepreneurs are subject to *financial frictions*, which may prevent the firm from producing at their optimal scale. Specifically, we assume a collateral constraint that limits the firm's debt-borrowing capacity to a multiple of the owner's assets, parameterized by λ. This constraint implies that firms' leverage, or debt to capital ratio, satisfies $(k_{it} - a_{it})/k_{it} \leq (\lambda - 1)/\lambda$.[20]

Given these assumptions, the profit maximization problem of an entrepreneur with assets a_{it} and a firm of quality z_{it} is given by

$$\pi(a_{it}, z_{it}) = \max_{k_{it}, l_{it}} \; y_{it} - Rk_{it} - wl_{it}$$

[18] Ideas are non-rival. However, they can be excludable under institutional arrangements such as patents. See Silveira and Wright (2010) and Akcigit, Celik, and Greenwood (2016) for the trade of ideas.

[19] Firms' intangible assets could include trademarks, patents, processes, permits, customer bases, business plans, or business knowledge. Consistent with our characterization of firms, Bhandari and McGrattan (2021) document that when a firm is sold, intangible assets account for 60% of the transacted price.

[20] This constraint can be micro-founded with an imperfect contract enforcement problem. As in most debt financing contracts, the firm cannot pledge the quality of the entrepreneurial project as collateral.

$$\text{s.t.} \quad y_{it} = z_{it} k_{it}^{\theta} l_{it}^{\nu}$$
$$k_{it} \leq \lambda a_{it} \tag{2}$$

where R is the capital rental rate. If the collateral constraint binds ($k_{it} = \lambda a_{it}$), the firm operates at a lower scale compared to the unconstrained profit maximization level.[21]

Public Firm As in Cagetti and De Nardi (2006), we assume that there is a second sector of production populated by a representative public firm. This aims to capture that, in the U.S. economy, around half of the total output is produced by publicly traded firms.

We assume that the public firm is owned by all households, in equal shares, and faces no financial frictions. The public firm is endowed with a constant return to scale technology

$$Y_{pt} = K_{pt}^{\eta} L_{pt}^{1-\eta}$$

where K_{pt} is the public firm's capital, L_{pt} its labor, and Y_{ct} its total output.

Financial Intermediary The financial intermediary takes deposits from households and rents capital to the firms at a price equal to the savings interest rate plus the depreciation rate: $R = r + \delta$. We assume that the representative intermediary operates in a perfectly competitive market and breaks even (i.e., makes zero profits). The resource constraint of the intermediary is

$$K_{pt} + \int k(a_{it}, z_{it}) \, \mathrm{d}N_{cm}^{e}(a_{it}, z_{it}) = \int a_{it} \, \mathrm{d}N_{cm}^{e}(a_{it}, z_{it}) + \int a_{it} \, \mathrm{d}N_{cm}^{w}(a_{it}, \varepsilon_{it}) \tag{3}$$

where N_{cm}^{e} and N_{cm}^{w} are cumulative distribution functions for entrepreneurs and workers, which are normalized such that $\int \mathrm{d}N_{cm}^{e} + \int \mathrm{d}N_{cm}^{w} = 1$. These measures correspond to the production stage after firm owners decide whether to be entrepreneurs or workers.

3.2 A Market for Firms

Firms are hard to valuate and price. This precludes the existence of a centralized market with a complete price schedule for different types of firms. Therefore, we model the market for firms using a search-theoretic approach characterized by bilateral random matching and *quid pro quo* trade. An interpretation of this setup is that agents can valuate only one firm at a time, which delays trade.

Trade in the market for firms consists of the transfer of both the firm's ownership and management in exchange for assets. Hence, the media of exchange in these transactions are the households' savings a. As firms are indivisible, when a buyer and a seller meet they only bargain over the selling price p.

[21] Appendix C.1 presents firms' input demand functions that characterize the static solution of (2).

Bilateral Meetings There are two types of meetings in the market for firms: *owner-owner* meetings and *owner-worker* meetings. We allow for different degrees of search frictions in each type of meeting. For a firm owner, an owner-owner meeting happens with probability α_o, and an owner-worker meeting happens with probability α_w. For a worker, an owner-worker meeting happens with probability α_w.

Note that firm owners are the only potential sellers, while both types of households can be buyers. This implies that in an owner-worker match, the owner is the potential seller, and the worker is the potential buyer. However, in the case of an owner-owner match, who is the buyer and who is the seller depends on the relative quality of the firms.

Let us first consider the owner-owner match and suppose that $z_{it} < z_{jt}$. Then, owner i with states $\mathbf{s}^o_{it} \equiv (a_{it}, z_{it})$ is the potential buyer, and owner j with states $\mathbf{s}^o_{jt} \equiv (a_{jt}, z_{jt})$ is the potential seller. This follows from the assumption that households can own only one firm at a time. Hence, no owner would buy a lower-quality firm. In this case, the total surplus from trading the ownership of firm z_{jt}, in exchange for p assets, is given by

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it} - p, z_{jt}) - W^o(\mathbf{s}^o_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{4}$$

where W^o and W^w are the value functions at the beginning of the production stage for firm owners and workers, respectively, which satisfy that $a_{it} - p \geq 0$ and $a_{jt} + p \geq 0$. As described below, T_{jt} is a utility transfer that sellers might receive that captures additional motives to trade firms. Upon selling, the household goes to the labor market with labor efficiency $\underline{\varepsilon}$, as presented in the first term of the seller's surplus.[22] The outside option for both agents (the terms with a minus in the surpluses) is the value of going to the production stage as firm owners with their initial states $\mathbf{s}^o_{it}$ and $\mathbf{s}^o_{jt}$, respectively.

Regarding the owner-worker match, suppose that firm's owner j with states $\mathbf{s}^o_{jt}$ meets with a worker i with states $\mathbf{s}^w_{it} \equiv (a_{it}, \varepsilon_{it})$. Then, the total surplus from trading firm z_{jt} is

$$\text{Total surplus} \equiv \underbrace{W^o(a_{it} - p, z_{jt}) - W^w(\mathbf{s}^w_{it})}_{\text{Buyer's surplus, } S_b} + \underbrace{W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) - W^o(\mathbf{s}^o_{jt})}_{\text{Seller's surplus, } S_s} \tag{5}$$

where the only difference relative to the previous match is the buyer's outside option. In this case, if the parties don't trade, the buyer would continue to the production stage as a worker with its initial state $\mathbf{s}^w_{it}$.

Alternative Motives Besides the purely financial reasons to trade firms studied in this paper, related to households' wealth, access to credit, and risk aversion, there could be

[22] Business owners that do not sell can shut down their firm and go to the labor market with the same labor efficiency $\underline{\varepsilon}$. This free exit assumption rules out the possibility of negative prices in our model.

other motives for why entrepreneurs sell their firms. Examples of these alternative motives to trade could include time-varying preferences (e.g., the non-monetary value of being self-employed) or motives related to owners' life cycle (e.g., health shocks or retirement). To account for these alternative motives to trade firms in a parsimonious manner, we assume that potential firms' sellers receive a *preference* shock κ_{jt} that captures additional benefits or a reduction in the opportunity cost, of selling their firm in the current period.[23] The preference shock follows

$$\kappa_{jt} = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\xi_{jt} \tag{6}$$

where $1 \leq \underline{\kappa} < \overline{\kappa}$, and the random variable ξ_{jt} is *iid* across time and firms and drawn from a Beta distribution with $\mathcal{B}(1, \beta_\kappa)$. We denote the CDF of κ as $\Psi(\kappa)$. Figure C.2 in the appendix exemplifies the distribution of preference shocks under different parameterizations.

The shock κ_{jt}, with domain in $[\underline{\kappa}, \overline{\kappa}]$, determines the additional utility transfer that the seller receives upon selling compared to the trading for a higher price $\kappa_{jt}p \geq p$ but no extra utility. Thus, for each potential seller j with states $\mathbf{s}^o_{jt}$, preference shock κ_{jt}, and price p, the utility transfer $T_{jt}(p) \equiv T(p; \mathbf{s}^o_{jt}, \kappa_{jt})$ is implicitly defined by

$$W^w(a_{jt} + \kappa_{jt}p, \underline{\varepsilon}) = W^w(a_{jt} + p, \underline{\varepsilon}) + T_{jt}(p) \tag{7}$$

which states that the seller is indifferent between selling at a higher price $\kappa_{jt}p$ with no transfer and the case with price p and receiving $T_{jt}(p)$. Hence, this utility transfer is similar in spirit to the classical Hicksian compensation. Intuitively, this transfer implies that the owner is willing to sell the firm at a $1-\kappa_{jt}^{-1}$ discount, relative to the full price $\kappa_{jt}p$. Thus, all else equal, higher values of κ_{jt} will make sellers willing to sell their firms at larger discounts and lower prices. In our quantitative application, we show that the preference shocks help us fit the trading rates of the largest firms. It is worth noticing that, through our analysis, we assume that the preference shocks are realized after agents meet and, hence, they occur after the identity of the potential buyer and seller is determined.[24]

Sufficient Condition for Trade Let $\underline{p}_{jt} \equiv \underline{p}(\mathbf{s}^o_{jt}, \kappa_{jt})$ denote the minimum price at which seller j is willing to sell its firm, i.e., the price at which the seller's surplus is equal to zero. Likewise, let $\overline{p}_{it} \equiv \overline{p}(\mathbf{s}_{it}, z_{jt})$ be the maximum price that buyer i is willing to pay for firm j, i.e., the price at which the buyer's surplus is equal to zero. A sufficient

[23]Preference shocks to buyers would play an equivalent role, altering prices due to their effects on buyers' surplus. For tractability, we assume that only sellers receive these shocks.

[24]While this assumption can be relaxed, doing so would complicate the characterization of the trades in the market for firms without additional insights.

condition for trade, meaning that there are positive gains from trading firm j, is that

$$\underline{p}_{jt} < \overline{p}_{it} \tag{8}$$

where the states of buyer i are $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, depending on the type of match (owner-owner or owner-worker, respectively). For a given meeting, condition (8) shows that the possibility of a trade is a function of the firms' potential sellers' and buyers' characteristics. In Section 5, we characterize the probability of buying and selling firms across agents' characteristics in our quantitative model.

Bargaining If there are positive gains from trade, we assume that the price is determined by a *Nash bargaining* protocol. Thus, the trading price p between buyer i with states $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$, and seller j with states $\mathbf{s}^o_{jt}$ and preference shock κ_{jt} solves

$$\begin{aligned} p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}) =& \arg\max_p \Big[S_b(\mathbf{s}_{it}, z_{jt}, p)\Big]^{\chi} \Big[S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p)\Big]^{1-\chi} \\ & \text{s.t.} \quad S_b(\mathbf{s}_{it}, z_{jt}, p) \geq 0,\ S_s(\mathbf{s}^o_{jt}, \kappa_{jt}, p) \geq 0 \end{aligned} \tag{9}$$

where S_b and S_s are the buyer and seller surpluses, defined in (4) and (5), and $0 \leq \chi \leq 1$ parameterizes buyers' bargaining power. Thus, if χ is near 0, the price will be close to the buyer's maximum price $\overline{p}_{it}$. Conversely, if χ is near 1, the price will be close to the seller's minimum price $\underline{p}_{jt}$. As we will explain in Section 4, information about the ratio of selling prices to firms' profits helps us identify this parameter.

3.3 Timing

The timing of the model can be summarized as follows: (1) Startup shocks, the quality of entrepreneurial projects z, and the labor efficiencies ε are realized. (2) Agents enter the market for firms (DM) where business owners can buy and sell firms, while workers can only buy. (3) Preference shocks κ are realized for potential sellers. (4) Agents enter the production stage (CM). Given prices and their current z, firm owners decide whether to operate the firm or go to the labor market. Finally, production occurs, and agents choose how much to consume and save.

3.4 Recursive Formulation and Competitive Equilibrium

To solve the model, we first recursively write the problem of firm owners and workers at the market for firms (CM) and at the production stage (DM), as shown in Appendix C.2. Appendix C.3 presents our model's formal definition of competitive equilibrium. We solve for the stationary competitive equilibrium of this model by approximating value functions using projection methods on a finite state space for which we solve all the possible matches and trading prices, as well as agents' and firms' optimal choices. See Appendix C.4 for a detailed description of our numerical solution.

4 Parameterization

This section describes our calibration strategy. We calibrate the model, at an annual frequency, to the year 2007. We focus on 2007 as that is the year we have both the SBO and SCF data available.

4.1 Assigned Parameters

We set the relative risk aversion parameter to $\sigma = 1.5$, the capital depreciation to $\delta = 0.06$, and the public's firm capital elasticity to $\eta = 1/3$. All three are common values in the literature. Regarding the preference shock κ, we set its domain to $[1, 3]$, which implies that sellers' have a maximum possible discount of 66% $(1 - 1/\overline{\kappa})$ coming from the preference shocks. Panel (a) of Table 2 summarizes these assigned parameters.

Table 2: Parameterization

Parameter	Value	Description
		(a) *Assigned Parameters*
σ	1.5	CRRA
δ	0.06	Capital depreciation rate
η	1/3	Capital elasticity
$\underline{\kappa}$	1	Preference shock, lower bound
$\overline{\kappa}$	3	Preference shock, upper bound
		(b) *Calibrated Parameters*
β	0.898	Discount factor
Υ	0.724	Curvature private firms technology
$(\lambda - 1)/\lambda$	0.397	Collateral constraint, maximum leverage
γ	0.930	Persistence private firm quality
ζ	0.939	1− Startup shock
z_{min}	1.118	Scale, z distribution
η_z	2.419	Shape, z distribution
ρ_ε	0.953	AR(1) parameter, ε distribution
σ_ε	0.240	Std. Deviation, ε distribution
$\mathbb{E}[\kappa]$	1.354	Preference shock, mean
α_o	0.803	Owner-owner | meeting probability
α_w	0.459	Owner-worker | meeting probability
χ	0.436	Buyers' bargaining power

4.2 Calibrated Parameters and Targeted Moments

We calibrate the remaining parameters such that the model replicates several key features of the U.S. economy, focusing on the trade of private firms. To reduce the parameter space dimension, we assume private firms' technology has the same relative elasticity between capital and labor as public firms. In such a way, a single parameter $\Upsilon < 1$ captures the degree of decreasing returns to scale in private firms' technology by setting $\theta = \eta\Upsilon$ and $\nu = (1 - \eta)\Upsilon$. After this, we have thirteen parameters, which we calibrate to match seventeen moments. Panel (b) of Table 2 presents these parameters with their calibrated values. We find those values by minimizing the distance between moments in the data and the model. Table 3 presents the seventeen moments we target in our calibration exercise. For an easier exposition, we divide these moments into five groups, which we now describe.

First, we target moments capturing the role of entrepreneurs in the economy. As reported in the 2007 SCF, we target that 6% of households are entrepreneurs, and they earn 20% of total income and hold 33% of the economy's wealth. Our second set of moments characterizes the distribution of income and wealth across all households and within workers and entrepreneurs. We target six different Gini indexes, which we also compute from the 2007 SCF. The table shows that our model matches the dispersion of income and wealth in the data very well. However, it slightly overpredicts the level of inequality among entrepreneurs. It is worth mentioning that different from the previous literature, which has abstracted from firm prices, our definition of wealth in the model includes the value of private firms $(a + p)$, consistent with the data. The parameters most informative about the income and wealth distribution in the model are the ones characterizing the distribution of firms' quality, z_{min} and η_z, and workers' labor efficiency, ρ_ε and ε.

The third and fourth sets of moments capture relevant characteristics of firms in the U.S. economy. First, we target a capital-output ratio of 3, which disciplines the discount factor β. Second, we target that private firms account for 50% of total output, which is consistent with the estimates in Dinlersoz et al. (2019), and lower than Asker, Farre-Mensa, and Ljungqvist (2014), which calculates that private firms account for 57% of total sales. Regarding private firms' leverage, we target our model's weighted average debt-to-capital ratio to be 0.35 to match private firms' aggregate debt-to-aggregate capital ratio reported by Bellon et al. (2023) using U.S. tax data. This moment pins down the collateral constraint parameter λ. We also target a firm-level exit rate of 0.09, which we computed from the Census Business Dynamics Statistics (BDS) for 2007. These moments are especially important to discipline the decreasing return to scale Υ and the parameters γ and ζ. It is worth mentioning that we calibrate a value of $\gamma = 0.93$ for the persistence of firms' quality, which is on the higher side relative to the literature.[25]

Our fifth and final set of moments captures relevant features of the trade of private firms documented in Section 2. This set includes four moments that discipline the four parameters of the market for firms. We target an annual trade rate of 3% and that workers purchase 66% of the firms. These moments are relevant for the search frictions parameters, α_o and α_w. Additionally, to identify the relevance of preference shocks, we target the 1.7% trade rate of the largest firms, defined by firms in the top output quartile. The preference shocks are particularly relevant for the trade of large and financially unconstrained firms, as the gains from trading these firms related to financial frictions are low. Indeed, we below show that excluding preference shocks significantly reduces the trade rates of the largest firms. From our calibration, we get that $\mathbb{E}[\kappa] = 1.354$, which implies an average discount of $\mathbb{E}[1-\kappa^{-1}]$=0.23.[26] Finally, we target a median price-to-profit ratio equal to 3.5, which we obtained from Dealstat.[27] This ratio is most informative for the buyers' Nash bargaining parameter χ. We obtain a value of $\chi = 0.442$, implying that sellers have a slightly higher bargaining power than buyers. Overall, Table 3 shows that our model does an excellent job matching the targeted moments. Especially the ones related to entrepreneurs, private firms, and the market for firms.

4.3 Other Untargeted Moments

A relevant feature of heterogeneous agents models with entrepreneurship is that they can replicate the income and wealth distribution observed in the data (Quadrini, 2000; Cagetti and De Nardi, 2006). This is possible thanks to the combination of uninsurable income risk and stochastic returns to wealth from entrepreneurial activity. Table C.2, in the Appendix, shows that this is also true in our model. Although we only targeted a set of Gini coefficients, the model does a good job matching the entire income and wealth distribution observed in the data. In Section 6 below, we test the predictions of our model against a battery of additional cross-sectional and longitudinal moments related to the market for firms that we did not target in our calibration exercise.

[25]For example, Buera, Kaboski, and Shin (2011) calibrate a value of $\gamma = 0.89$. As shown in Moll (2014), the persistence of firms' productivity in this class of models is crucial in determining the strength of the self-financing channel and, hence, the capital misallocation losses from financial frictions. In our model, the option to buy and sell firms non-trivially affects entrepreneurs' saving motives. For example, while the option to sell might discourage entrepreneurs from growing out of their constraints, the option value to buy better firms might strengthen saving motives. In our quantitative exercise below, we find that the market for firms plays a significant role despite the strong self-financing channel in our parameterization.

[26]We directly target the mean of κ, which implicitly defines the parameter β_κ. In detail, note that $\mathbb{E}[\kappa] = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\mathbb{E}[\xi]$ and $\mathbb{E}[\xi] = \frac{1}{1+\beta_\kappa}$, which defines β_κ given $\underline{\kappa}$, $\overline{\kappa}$ and $\mathbb{E}[\kappa]$. Figure C.2 shows the distribution of the preference shocks, fixing the upper and lower bounds, for different values of $\mathbb{E}[\kappa]$.

[27]Dealstat (formerly Pratt's Stats) is a database of business transactions. We use their publicly available reports to compute an average median selling price to EBITDA ratio of 3.5 from 2010 to 2018 in the US.

Table 3: Targeted Moments

	Source	Data	Model
Entrepreneurs			
Fraction of entrepreneurs	SCF	0.06	0.06
Income share of entrepreneurs	SCF	0.20	0.21
Wealth share of entrepreneurs	SCF	0.33	0.38
Income and Wealth Distribution			
Gini income, all households	SCF	0.62	0.61
Gini wealth, all households	SCF	0.82	0.83
Gini income, entrepreneurs	SCF	0.67	0.77
Gini wealth, entrepreneurs	SCF	0.74	0.81
Gini income, workers	SCF	0.58	0.56
Gini wealth, workers	SCF	0.78	0.79
Private and Public Firms			
Capital to output ratio	See text	3.0	3.0
Private Firms			
Output share	See text	0.50	0.45
Leverage	See text	0.35	0.35
Exit rate	BDS	0.09	0.09
Trade of Private Firms			
Trade rate, all firms	SBO	0.030	0.031
Trade rate, largest firms	SBO	0.017	0.013
Share purchased by workers	SBO	0.66	0.67
Median price/profits	DealStats	3.5	3.3

Notes: Data moments correspond to the year 2007. Wealth in the model is defined as the sum of the risk-free asset and the value of the firm $a + p$. Trade rate, largest firms is the trading frequency of firms in the top quartile of the output distribution.

5 Model Properties

This section describes the main properties and workings of our model. First, we discuss and quantify the different motives behind firm trade. Second, we characterize the price at which firms trade and who buys and sells firms in our economy. Finally, we describe the implications of this market for firm dynamics and capital allocation.

5.1 Motives for Trading Firms

Exchanges in the market for firms are voluntary. Hence, a necessary condition for gains from trade is that agents have different valuations for the same firm. In particular, the buyer must have a higher valuation than the seller. In our theory, given the agents' outside options, heterogeneous valuations for firms arise from three sources: the preferences shocks, firms' credit constraints, and incomplete markets. We now describe and quantify each of these three motives behind the trade of firms.

Table 4: Trade Rate Decomposition

	All Firms	
	Trade rate	Relative
Baseline	3.1%	1.00
No preference shocks	2.1%	0.69
No collateral constraint	1.0%	0.32
No preference, no collateral	0.4%	0.13

Notes: Steady-state comparisons of the market for firms' trade rate under different parameterizations. Relative is the ratio of each trade rate to the Baseline model. No preference shocks turn off the alternative motives to trade firms by setting $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$. No collateral constraint assumes $\lambda \to \infty$. No preference, no collateral considers both previous cases simultaneously.

Preference Shocks We introduce alternative motives to trade firms through sellers' κ shocks at the beginning of the market for firms. These shocks aim to capture, parsimoniously, all the motives to trade firms unrelated to the financial channels we study in this paper. To evaluate the role of preference shocks, the second row of Table 4 presents the firms' trade rate when we turn off these alternative motives. This comparative static exercise sets $\mathbb{E}[\kappa] = 1$ and $Var[\kappa] = 0$ while keeping the rest of the parameters fixed. Without preference shocks, the economy's annual firms' trade rate falls from 3.1% to 2.1%, showing that preference shocks explain around 31% of the transactions in the market for firms. Figure C.1 in the Appendix reveals that excluding preference shocks leads to a disproportionately large drop in the trade of large and low ARPK firms. Hence, as that figure shows, preference shocks play a relevant role in the trade of large and unconstrained firms, for which the gains from trade due to financial frictions are small.

Credit Constraints Regarding the financial motives to trade firms, we first focus on the role of credit constraints. This channel arises from the collateral constraint in entrepreneurs' problem, presented in (2), that restricts firms' capital to a multiple λ of their owners' wealth. Consequently, whenever an entrepreneur is credit-constrained, a wealthier buyer can obtain a higher profit stream out of the same firm as it would be able to operate closer to its optimal scale, generating potential gains from trade. To quantify the importance of this channel, we set $\lambda \to \infty$, which implies that the firms' profits stream is no longer a function of their owners' wealth. The third row of Table 4 shows that removing credit frictions significantly reduces the frequency of trades in the market for firms to 1.0%, indicating that the bulk of the transactions in our baseline economy, 68%, are driven by credit constraints. As we show below, this channel plays a crucial role in the ability of our model to match the fact that younger, smaller, and higher ARPK have the highest trading rates, documented in Figure 1, as these are the firms more likely to be credit constrained in our model economy.

Risk and Incomplete Markets Risk aversion and incomplete financial markets constitute the third motive for trading firms. In our model, owning and operating a firm is associated with uninsurable income risk as the firm's quality z is stochastic. Thus, even without credit constraints, agents can have different valuations for the same firm as a function of their wealth. For low-wealth owners, selling the firm allows them to front-load consumption and achieve an earlier risk resolution. For high-wealth owners, consumption is less dependent on shocks to the firm's profits. In other words, the covariance between their stochastic discount factor and the realization of profits is small, increasing their ability to bear risk. Therefore, the value of owning a firm will vary across the wealth distribution, generating potential gains from trade. To evaluate the importance of this channel, we turn off both the preference shocks and firms' credit constraints. The last row of Table 4 shows that, in this case, the trade rate is 0.4%. This result suggests that risk and incomplete markets account for 13% of the firms' trades in our baseline economy.

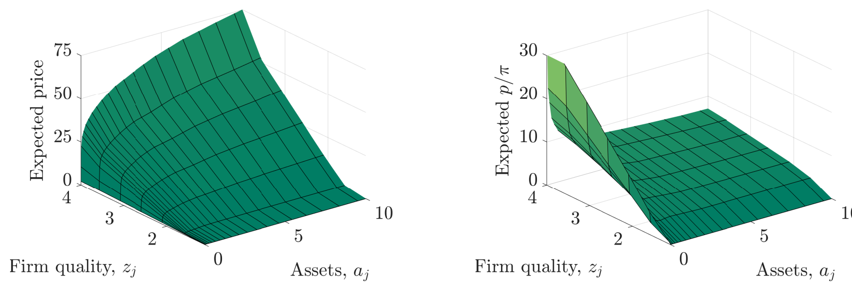
5.2 Who Buys and Who Sells Firms?

We now describe typical buyers and sellers in the market for firms. We start our characterization by analyzing the equilibrium prices at which firms trade. Panel (a) of Figure 5 presents the average price $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})]$ resulting from the Nash bargaining protocol in the sellers' state space $\mathbf{s}^o_{jt} = (a_j, z_j)$, after integrating over the preference shock κ_j and all potential buyers $\mathbf{s}_{it}$. As expected, selling prices are increasing firm quality z_j. However, due to the collateral constraint on owners' wealth and incomplete markets, holding the firm's quality fixed, the price is increasing in the owners' assets a_j. Note that firms' prices would be unrelated to the current owner's wealth under perfect credit markets. Thus, due to imperfect credit markets, high-quality firm owners with low wealth will be willing to sell their firms at a relatively low price as it will take time and high saving rates to grow out of their borrowing constraint through self-financing. Nevertheless, as panel (b) of Figure 5 shows, these transactions have considerably high price-over-profit ratios, which illustrates the small scale of operation of constrained and low-wealth owners with high-quality firms relative to the price at which they can sell their business.[28]

[28] According to Dealstat, the median price-over-profit ratio in the Information sector equals 9, considerably higher than the economy-wide 3.5 number. This evidence is consistent with our model's large price-over-profit ratios for high-growth potential firms.

Figure 5: Prices in the Market for Firms

(a) Avg. Price (b) Avg. Price/profits

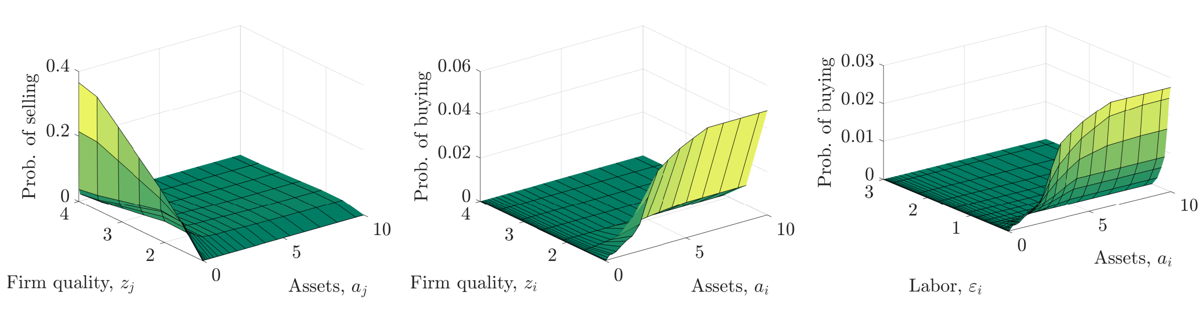


Notes: Average price $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})]$, and price over profits $\mathbb{E}_{\mathbf{s}_{it},\kappa_{jt}}[p(\mathbf{s}_{it}, \mathbf{s}^o_{jt}, \kappa_{jt})/\pi(\mathbf{s}^o_{jt})]$, in owners' state space, $\mathbf{s}^o_{jt} = (a_{jt}, z_{jt})$, after integrating over shocks κ_{jt} and trading counterparts, $\mathbf{s}_{it} \in \{\mathbf{s}^o_{it}, \mathbf{s}^w_{it}\}$.

Considering how trading prices are determined, we characterize who buys and sells firms in our economy. Panel (a) of Figure 6 presents the probability that a firm owner sells its firm in the (a_j, z_j) space. The figure shows that owners with low wealth and high-quality firms have the highest probability of selling. In those cases, there will be high gains from trade as the current owner lacks the assets to operate at the optimal scale. Panels (b) and (c) present the probability of buying a firm for firm owners in the (a_i, z_i) space and for workers in the (a_i, ε_i). These panels show that the probability of buying is the opposite mirror image of the likelihood of selling. Thus, firms' buyers are mostly wealthy households that currently own low-quality firms (low z) or wealthy workers with low labor efficiency (low ε). In Section 6.1, we show that our model's prediction that business buyers are wealthier than the average household is consistent with the data.

Figure 6: Buyers and Sellers in the Market for Firms

(a) Pr. of selling (b) Pr. of buying, owners (c) Pr. of buying, workers



Notes: Probabilities (Pr.) of trade after integrating over preference shocks and trading counterparts.

Overall, the previous results indicate that typical sellers in our economy are firm owners with high-quality firms but low wealth, and typical buyers are wealthy agents with relatively low-quality firms or low labor efficiency. Thus, as we show below, these trades in the market for firms between constrained and potentially unconstrained owners lead to a better allocation of productive projects and available resources in the economy.

6 Financial Frictions as a Motive to Trade Firms

Before moving on to our aggregate results, this section presents three exercises that evaluate testable predictions of our theory about financial frictions being a relevant motive to trade firms. In the first two exercises, we compare our model predictions about buyers' and firms' characteristics in the cross-section to those documented in Section 2.1. In our third exercise, we simulate a panel of firms and compare the post-trade firm dynamics implied by our model to those documented in Section 2.2. Finally, we discuss alternative theories of firms' trade and compare their predictions to the data.

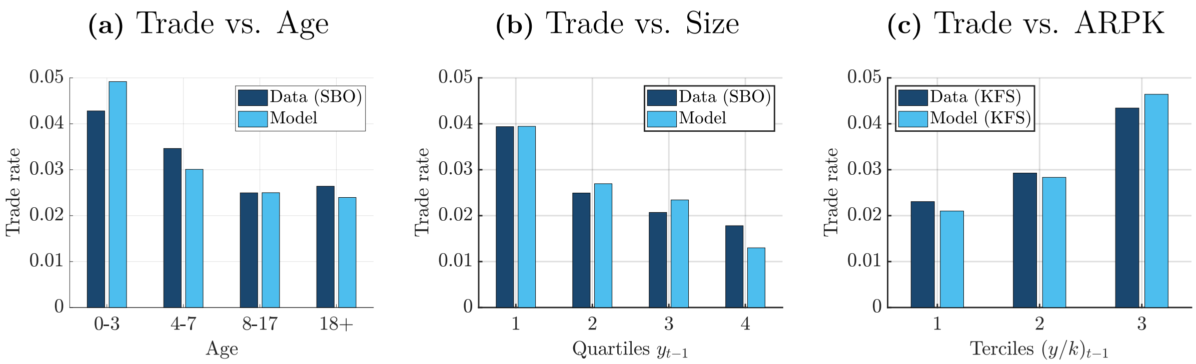
6.1 Buyers' Wealth in Data and Model

We first test the prediction of our model about buyers' wealth. As we showed in Section 5.2, if financial frictions drive firms' trade, business buyers will be, on average, wealthier than sellers. Although we do not observe the wealth of the agents in each transaction, we can measure the wealth of the average buyer in the SCF. In Section 2.1.3, we documented that the average firm buyer is wealthier than the average household but less so than the average entrepreneur. We compute analogous moments in our model. Table C.3 shows that our model aligns remarkably well with the data despite these moments not being targeted in our calibration. Including business wealth $(a+p)$, our model predicts that buyers are 3.1 times wealthier than the average household, while this number is 3.8 in the data. Excluding business wealth (a), this ratio equals 2.7 in the data and the model. Our model is also consistent with the wealth ratio of buyers to the average entrepreneur, which is 0.69 for total wealth and 0.79 excluding business wealth (0.54 and 0.75 in the model). Thus, while firm buyers are wealthy, they are less so than incumbent entrepreneurs who have previously accumulated wealth. Overall, these results about buyers' characteristics are consistent with the financial motives to trade firms in our model.

6.2 Trade Rate and Firms' Characteristics in Data and Model

Second, as described throughout the paper, if financial frictions are an important reason for trade, credit-constrained firms should be more likely to be bought and sold as gains from trade are the highest for those firms. We test this first prediction of the model by analyzing the relation between trade and firms' observable characteristics. As in the empirical section, we consider two commonly used proxies of credit constraints: firms' age and size, as younger and smaller firms are more likely to be financially constrained. In addition, we analyze firms' ARPK since credit-constrained firms will have high capital returns but cannot increase their investment.

Figure 7: Trade Rate by Firms' Characteristics in Data and Model



Source: SBO, KFS, and model simulated data.
Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 1 for a description of the data moments.

Following the analysis in Section 2.1.4, we simulate data from our model and compute the trade rate conditional on firms' characteristics. Figure 7 shows that consistent with the data, our model predicts that younger, smaller, and higher ARPK firms exhibit the highest probabilities of trade. It is important to emphasize that these relations were *not* targeted in our calibration exercise. Instead, they result from the key prediction of our theory that credit-constrained firms are the ones more likely to be traded and that these characteristics are strongly correlated with binding credit constraints in our model.

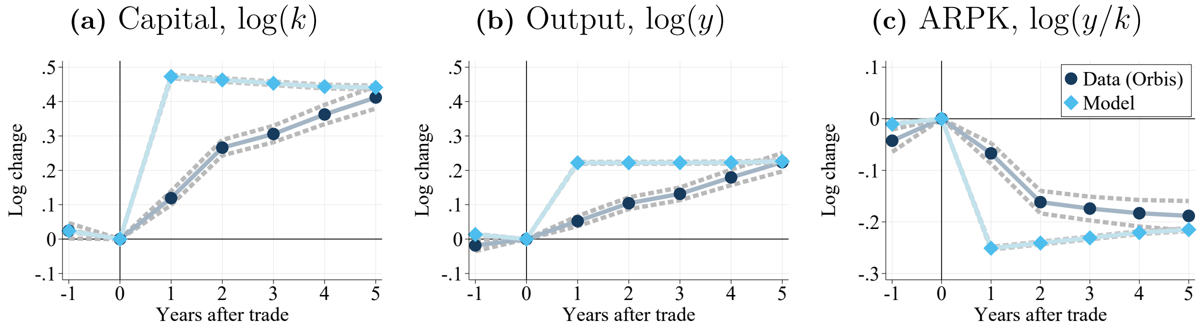
6.3 Post-Trade Firm Dynamics in Data and Model

We now compare the firm dynamics after trade in our model to those documented in Section 2.2 for firms' output, capital, ARPK, ARPL, profitability, and leverage.[29] To study the post-trade dynamics implied by our quantitative model, we generate a panel of 2.5 million firm-year observations and estimate regression (1) or each variable of interest using the model simulated data. In addition to our quantitative analysis, Proposition 1 part (1) in Appendix C.5 analytically characterizes the post-trade dynamics of these six variables when financial frictions drive trade in the market for firms.

We first analyze the joint dynamics of capital and output. Our theory predicts that firms' trade alleviates financial constraints. Hence, before being traded, firms operated with lower capital and higher ARPK relative to their unconstrained level. After a trade,

[29]This exercise implicitly assumes that the post-trade firm dynamics in high-income Europen countries are informative about those in the U.S., the economy for which we calibrate our model. Consistent with this assumption, Kochen (2023) shows that key firm dynamics moments related to firm exit and growth rates over the life cycle of firms in this group of high-income countries are similar to those documented for the U.S. by, for example, Haltiwanger, Jarmin, and Miranda (2013).

Figure 8: Capital, Output, and ARPK Dynamics After Trade in Data and Model



Source: Orbis Historical and model simulated data.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

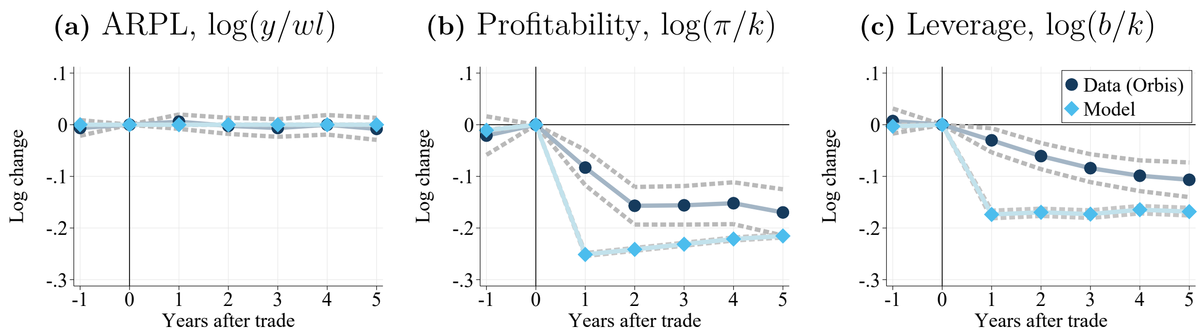
firms' capital increases, and crucially, capital increases more than output, reducing firms' ARPK over time. Figure 8 shows that although the dynamics in our model are faster, with firms immediately jumping closer to their optimal scale, the overall effect five years after trade in our model aligns exceptionally well with the evidence documented in Section 2.2.[30] Panel (a) and (b) report that firms' capital grows twice as much as output, by 0.44 log points after five years in our model and 0.41 log points in the data. As Panel (c) shows, this results in a significant reduction in firms' ARPK of -0.21 log points in the model and -0.19 log points in the data five years after trade.

In addition to the dynamics of capital, output, and ARPK, we test the predictions of our model for the evolution of firms' ARPL, profitability, and leverage. Figure 9 presents the post-trade dynamics of these variables in our model and the data. Panel (a) shows that firms' ARPL in our model is constant, as we assume that financial frictions do not distort labor input decisions. This assumption aligns well with the data, as firms' ARPL is remarkably stable after trade, and the change is not statistically different from zero. Panel (b) shows that firms' profitability, measured by profits over capital, decreases in our model, which is consistent with the data. Credit-constrained firms in our model have a distorted profit-to-capital ratio, which is higher than the optimal unconstrained level. Thus, similar to the ARPK dynamics, capital grows more than profits after trade, reducing firms' profitability by -0.21 log points five years after trade in the model, which aligns well with the -0.17 log points change in the data. Finally, panel (c) shows the post-trade dynamics of firms' leverage in the data and those implied by our model. As firms' buyers tend to be wealthier than sellers in our model, most of the additional capital comes from owners' equity, which results in a significant decrease in firms' leverage of -0.16 log

[30]The absence of other frictions in our framework, such as capital adjustment costs, can partly explain the faster post-trade dynamics in our model relative to the data.

points five years after trade. The reduction in leverage implied by our model aligns well with the data, where leverage falls by -0.11 log points after five years.[31]

Figure 9: ARPL, Profitability, and Leverage Dynamics After Trade in Data and Model



Source: Orbis Historical and model simulated data.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

It is important to mention that our model qualitatively matches all these cross-sectional and longitudinal facts exceptionally well, even though we did not directly target these moments in our calibration exercise.[32] Rather, all these patterns predicted by our model arise from the key endogenous motive to trade firms we study in this paper arising from the presence of financial frictions in the economy.

6.4 Alternative Motives to Trade Firms

Before continuing to our aggregate quantitative results, in this section, we discuss some alternative motives for trading firms and compare their predictions to the data.

Owners' Life Cycle One first potential motive, captured in our framework through the exogenous preference shocks, relates to owners' life cycle (e.g., health shocks or retirement). Panel (b) of Figure A.5, in the appendix, shows the vast majority of firm sellers are young- to middle-aged entrepreneurs, suggesting that sellers' retirement motives explain a small fraction of the trades in the market for firms. The preference shocks in our model also capture other preference-related motives to sell firms, such as varying tastes for self-employment (Mahone, 2023).

Managerial Abilities Another potential motive is related to entrepreneurs' heterogeneous managerial abilities (Holmes and Schmitz, 1995; Caselli and Gennaioli, 2013) or

[31]The post-trade dynamics of capital and debt documented in Section 2.2 indicate that firms receive sizable equity injections from their new owners. This evidence is consistent with our model, where gains from trade come from differences in wealth and not necessarily from buyers having more access to debt financing than sellers (higher λ in our model), which would predict an increase in leverage after trade.

[32]As Table 3 reports, we only target four moments about the market for firms: the aggregate trade rate, the largest firms' trade rate, the share purchased by workers, and the median price-to-profits ratio. Regarding the role of fin

differences in abilities between creating new projects and managing firms (Silveira and Wright, 2010). According to this motive, trades in the market for firms lead to a better allocation between managers and firms. As we formally argue in Proposition 1 part (2), in Appendix C.5, if business buyers have a higher managerial ability than sellers, we should see an increase in firms' profitability after trade, which is at odds with data. As Table C.1 summarizes, this proposition also shows that this motive would imply counterfactual post-trade dynamics for firms' ARPK and leverage. Thus, we do not find evidence supporting motives related to heterogeneous managerial abilities behind firms' trade.

Span-of-Control A third potential motive to trade could be related to heterogeneity in entrepreneurs' span-of-control (parameterized by $\Upsilon = \theta + \nu$ in our model), where gains from trade in the market for firms would come from transactions between low to high span-of-control entrepreneurs. Proposition 1 part (3) in Appendix C.5 shows that if the buyer has a higher span-of-control than the seller, this motive predicts a decrease in firms' ARPK and ARPL. While ARPK does fall, this prediction is inconsistent with the data as the change in firms' ARPL is not statistically different from zero in the years after trade. Furthermore, if buyers and sellers have the same wealth, this motive would predict that firms' leverage rises after trade, which is counterfactual. Hence, the post-trade firm dynamics implied by the heterogeneous span-of-control motive are at odds with the data.

To summarize, this section shows that the main testable predictions of our theory about the cross-sectional characteristics of buyers and firms and regarding post-trade firm dynamics are consistent with the data. While the evidence discussed in this section does not rule out that other motives could play a role in the market for firms, it is informative about their relevance. In this sense, this evidence puts a high bar for alternative theories for the trade of firms, as they would need to account for the cross-sectional and longitudinal facts we have documented in this paper. After showing that the evidence is consistent with financial frictions being an important motive for trading firms, in the next section, we use our model to quantify the role of the market for firms in the macroeconomy.

7 Macroeconomic Implications

This section presents our main quantitative exercises. First, we perform two counterfactual experiments that quantify the relevance of the market for firms as a mechanism through which entrepreneurial projects and available resources are allocated in the economy. Second, we study the level of TFP predicted by our model across economies with different degrees of financial development and functioning market for firms.

7.1 The Role of the Market for Firms

We now consider two counterfactual experiments that quantify the importance of the market for firms. Both experiments consist of steady-state comparisons of our model under different parameterizations. In the first experiment, we take our baseline model and analyze the implications of a partial or total market shutdown. In the second experiment, we analyze the level of external financing that an alternative economy with no trade of firms requires to match the TFP level of our baseline economy.

7.1.1 Closing the Market

Table 5 presents the results of our first counterfactual experiment. As a reference, the first column of the table has some relevant moments of our baseline economy. The second and third columns report the percentage change when the market for firms partially and then completely shut down. In both cases, we only vary the search frictions' parameters in the market for firms, α_o and α_w, while maintaining the rest fixed. For the partial shutdown case, we divide in half both parameters such that their relative values are the same and, hence, the fraction of firms purchased by workers is unchanged. For the complete shutdown case, we set both parameters equal to zero.

Table 5: Closing the Market for Firms

		Δ %	
	Baseline Economy	Partial $(\alpha_o, \alpha_w)/2$	Total $(\alpha_o, \alpha_w) = \mathbf{0}$
Fraction of entrepreneurs	0.06	-2.4%	-4.5%
Private firms output	0.57	-4.8%	-9.1%
Private firms TFP	1.17	-1.2%	-2.2%
Exit rate	0.09	-10.2%	-27.5%
Public firms output	0.71	2.6%	5.1%
Total output	1.29	-0.7%	-1.3%

Notes: The Partial column presents the results for the market partial shutdown, obtained dividing by the half the parameters α_o and α_w. The Total column presents the results when both parameters are equal to zero, thus a total market shutdown. TFP is measured as $Y_e/(K_e^\theta L_e^\nu)$, where $(.)_e$ denotes the aggregate variables of the entrepreneurial sector.

In both cases, private firms' output considerably falls by -4.8% and -9.1% for the partial and the complete shutdown case, respectively. For easiness in the exposition, we focus on the total shutdown results. The remaining rows of Table 5 show that both extensive and intensive margins explain the fall in entrepreneurial output. First, regarding the extensive margin, the share of active entrepreneurs falls by 4.5%. Additionally, without the market for firms, the entry and exit rate into entrepreneurship significantly decreases by 27.5%. Regarding the intensive margin, the remaining private firms exhibit a poorer allocation of capital and firms' qualities, as shown by the entrepreneurial TFP, which decreases by 2.2%. Total output in the economy also decreases, but to a lower extent, by 1.3%. General equilibrium effects and the assumption that the production of private and public firms are perfect substitutes explain the smaller aggregate effect. Indeed, an increase in the production of the public firm of 5.1% partially offsets the fall of entrepreneurial output.

7.1.2 Baseline vs. No Market Economy

From the firms' perspective, rather than the owners', the market for firms serves as an alternative source of financing. To better understand its importance, in our second exercise, we quantify the additional debt financing that an economy without trading firms would require to achieve the same allocative efficiency as our baseline economy with the market for firms. To do this, we compare steady states of a "no market economy", with $\alpha_o = \alpha_w = 0$, against our baseline model under different credit frictions. As Figure C.3 in the appendix shows, private firms' aggregate leverage and TFP are increasing in the degree of financial development, governed by the parameter λ. Higher λ implies easier access to credit as entrepreneurs can borrow more with the same level of assets. This figure shows, however, that for the same λ the no market economy attains a lower TFP level than our baseline model. This result is explained by the more severe misallocation between entrepreneurial projects and available resources when the market for firms is absent.

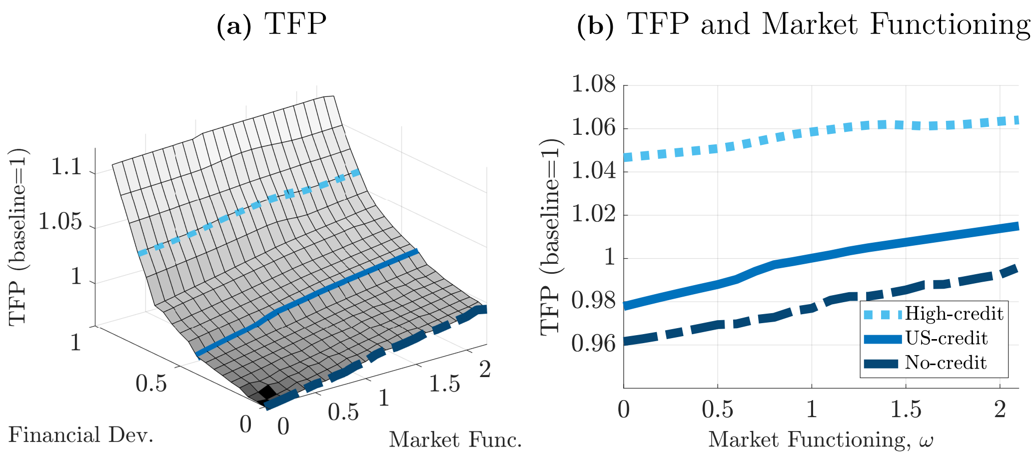
We then *ask*: what credit conditions does the no market economy require to match the TFP level of our baseline economy? Using Figure C.3, we can identify the level of λ such that the no market economy attains the same TFP as the baseline. These panels show that the no market economy requires an increase in private firms' debt financing such that the aggregate debt-to-capital ratio rises by 14 p.p., from 0.35 to 0.49, a significant increase. To put this number in perspective, private firms' leverage in the U.S. dropped by 5 p.p. during the Great Recession (Bellon et al., 2023). Thus, this exercise shows that the better allocation of resources thanks to the markets for firms is equivalent to an expansion of debt financing that leads to a 14 p.p. rise in private firms' aggregate leverage.

Altogether, these two counterfactual exercises demonstrate that the market for firms is a quantitatively relevant mechanism through which entrepreneurial projects and available resources can be better allocated in the economy.

7.2 Financial Development and the Market for Firms

In our model, the functioning of both markets for credit and firms determines the capital allocation in the economy. In this final section, we study the interaction between these two markets and their implications for aggregate productivity. Figure 10 shows private firms' TFP, relative to the baseline, for economies with different degrees of financial development and functioning markets for firms. We parameterize financial development by firms' maximum leverage defined by $(\lambda - 1)/\lambda$. We vary the functioning of the market for firms through different values of a parameter ω that scales the meeting probabilities: $\alpha_o(\omega) = \min\{\omega\alpha_o, 1\}$ and $\alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$ where (α_o, α_w) are the values of the baseline calibration. Thus, $\omega = 0$ corresponds to the total market shutdown, previously analyzed, and $\omega = 1$ to the baseline parameterization. Figure C.4 in the appendix shows similar results for entrepreneurial output and firms' trade rates.

Figure 10: Financial Development and Functioning of the Market for Firms



Notes: Financial Development is parameterized by firms' maximum leverage, $(\lambda - 1)/\lambda$. Market Functioning is parameterized by ω multiplying the search frictions in the market for firms $\alpha_o(\omega) = \min\{\omega\alpha_o\}, \alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$. Panel (a) plots private firms' TFP in the financial development and functioning of the market for firms' space. Panel (b) plots TFP in the market functioning space for three different levels of financial development: High-credit, US-credit, and No-credit with $(\lambda - 1)/\lambda$ equal to 0.75, 0.397, and 0. The baseline calibration corresponds to the case $(\lambda - 1)/\lambda = 0.397$ and $\omega = 1$.

Panel (a) of Figure 10 shows that TFP is increasing in both financial development and the functioning of the market for firms. In particular, note that for the case of $\omega = 0$ (No Market economy), our model implies that higher levels of financial development lead to a better allocation of capital and higher TFP, as in the finance and misallocation literature (Buera, Kaboski, and Shin, 2011; Midrigan and Xu, 2014; Moll, 2014). However, unlike previous papers in this literature, this figure also shows that for any given level of financial development, aggregate TFP can increase through a better-functioning market for firms.

To better make this point, panel (b) of Figure 10 presents three hyperplanes in the ω-space considering different levels of financial development: High-credit, US-credit, and No-credit (maximum leverage of 0.75, 0.397, and 0, respectively). There are two main takeaways from this panel. First, in a high-credit economy, the TFP gains from a better-functioning market for firms are limited, as shown by the flatter slope in the top line of panel (b). This result is because, in higher-credit environments, firm owners can produce closer to their optimal scale through debt financing, which reduces the gains from trading firms. Second, economies with less-developed financial markets can achieve TFP levels closer to an economy with US-credit through a better-functioning market for firms. Thus, the market for firms can substitute for debt financing in less developed credit markets. For example, the No-credit economy can attain the same level of TFP as the baseline calibration with twice as large search frictions parameters (ω above 2), with a trade rate in the market for firms of 5% as panel (b) of Figure C.4, in the appendix, shows.

The empirical evidence of firm dynamics after trade documented in Section 2.2 is consistent with these quantitative results. The fact that post-trade firm growth is twice as large in middle-income and less financially developed countries than in high-income economies is in line with the prediction of our model that the gains from trading firms are higher in environments with tighter financial frictions. These results demonstrate that the market for firms can play an even more important role as a capital allocation mechanism in economies with less developed financial markets.

8 Conclusions

We use microdata from business owners, households, and firms to provide novel cross-sectional and longitudinal facts about the market for firms. We document that one out of four entrepreneurs purchased their business, with younger, smaller, and higher ARPK firms having the highest trading rates. After trade, firms experience substantial capital and output growth, with capital outpacing output, significantly reducing firms' ARPK. Firms' ARPL remains constant after trade, while profitability and leverage decrease.

We explain these empirical findings by developing a general equilibrium model of entrepreneurship and frictional trade of firms. By introducing financial frictions as a micro foundation that generates gains from trade, our model can account for the empirical patterns. It accounts for the cross-sectional facts, as younger, smaller, and higher ARPK firms are more likely to be financially constrained. Furthermore, it accounts for the longitudinal facts as firms' trade alleviates financial constraints in the model.

Our quantitative results imply that firms' trade significantly improves the allocation of capital and productive projects in the economy. We argue that the market for firms can play an even more important role in economies with tighter credit frictions, as the potential gains from trade are higher. The observation that post-trade firm dynamics are twice as large in middle-income and less financially developed countries is consistent with this prediction. Our results suggest that a promising avenue for future work is to understand better how policies can improve the functioning of the market for firms, especially in economies with less developed financial markets.

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Online Appendix for
Financial Frictions and the Market for Firms

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Table of Contents

A U.S. Data Appendix

This appendix describes our primary data sources, presents robustness, and additional exercises about the market for firms in the US economy.

A.1 Data Sources

A.1.1 Survey of Business Owners (SBO) - PUMS

The SBO is a comprehensive survey of firms and firm owners in the U.S. The PUMS sample is representative of non-farm private businesses with receipts of $1,000 or more and is available for the year 2007. The SBO is conducted at the company or firm-level. A company is a business consisting of one or more domestic establishments. The survey is designed to identify the ultimate owners of firms and their characteristics.

Table A.1 reports the total number of owners and firms in the SBO. From those, we first restrict to the owners who report how do they acquire their business. The SBO already restricts to self-employed business owners, thus for our definition of entrepreneurs, we just have to restrict to business owners who actively manage their firm. Our baseline sample consist of almost 700,000 entrepreneurs which own around 500,000 different firms.

Table A.1: 2007 SBO Sample

	#Dropped	#Owners	#Firms
All	-	3,409,393	2,165,680
Report Acquisition	1,244,852	2,164,541	1,291,292
Manage	1,052,287	1,112,254	841,254
Employer firm	413,603	698,651	501,564

From this survey we mainly focus on how the owners acquired their firms. In addition, we use information on the characteristics of the firm (established year, employment, payroll, receipts, sector, location, operation status, number of owners) and of the owners (age, acquisition year, ownership percentage, education level, previous occupation). We use this information to do a thorough characterization of the trade of firms.

Using the SBO we can also obtain information on firms and owners close to the time at which the firm was traded. To study firms' and buyers' characteristics *when purchased* we look at owners that acquired the firm through a purchase in the same year of the survey. Furthermore, the SBO provides information on firms' and owners' characteristics for those owners who report an exit because they sold their firm in the year of the survey.

We use this information to characterize firms and their previous owners *when sold*. For all our calculations we use the sample weights provided by the survey.

A.1.2 Survey of Consumer Finances (SCF)

The SCF is a household-level survey that includes extensive information on households' income, balance sheets, and demographic characteristics. The public microdata is available every three years for the period 1989-2016.

Table A.2: 1989-2016 SCF Sample

	#Dropped	#Households
Income and wealth		
All	-	47,769
$21 < \text{age} < 78$	3,528	44,241
Positive income	67	44,174
Firm acquisition		
Manage and own	35,468	8,706
Employer firm	1,379	7,327

In the SCF we identify entrepreneurs as those households whose household head: is self-employed, owns a business, and has an active management role in it. The SCF also provides information of privately held businesses which are actively managed. Business owners can report information for up to three or two firms, depending on the survey year. For our baseline calculations we focus on the characteristics of the main business, defined as the one with higher reported value. Using this information, we can identify the entrepreneurs that own a firm with a positive number of employees.

Table A.2 reports our sample selection criteria and the number of households in our SCF sample. For our calculations of the moments of income and wealth we restrict to a sample of households whose household head is between 22 and 78 years old and have a positive income. For our calculations of the trade of firms trade we focus on entrepreneurs, which considering our baseline definition (with employer firms), are 7,327 households between 1989 and 2016, which is a significantly smaller than the one in our SBO sample.

In addition to the information on entrepreneurs and how do they acquired their firm, we use the SCF to compute relevant moments from the income and wealth distribution in the U.S. economy. Our measure of household wealth is the variable constructed by the

Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real estate, the value of farm and private businesses owned by the household. Debt includes both housing debt (mortgages), debt from lines of credit and credit cards, and installment loans.

Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non-classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities. For all our calculations we use the sample weights provided by the survey.

A.1.3 Kaufman Firm Survey (KFS)

The KFS is a panel survey that tracks almost 5,000 business that start their operations in 2004 through 2011. The initial sample was created by using a list frame sample of start-up businesses from the Dun & Bradstreet Corporation (D&B) database. The KFS collects information from business' and owner's characteristics and, in particular, they provide information about firms' balance sheets.

Table A.3 shows the sample selection. Following the previous literature, we drop firms that at some point refuse to answer and observations with missing values of employment, revenues, sales, assets, cash, and accounts receivable. Our baseline sample remains with 2,841 firms and 13,457 observations (firm×year).

Table A.3: 2004-2011 KFS Sample

	#Dropped	#Owners	#Firms
All	-	39,424	4,928
Answer	13,624	25,800	3,225
Missing	16,684	9,116	2,366

We define capital as total assets without cash holdings and accounts receivable. Total assets is composed by product inventories, land and buildings and structures, vehicles, equipment/machinery, other properties, cash, and other. To approximate the capital returns we consider the average revenue product of capital (ARPK) measured as firms' revenue to capital ratio. In the KFS we identify trades through exits of owners that report having sold or merged their business. For all our calculations we use the sample weights provided by the survey.

A.1.4 Annual Survey of Entrepreneurs (ASE)

The ASE is a representative sample of all non-farm businesses filing Internal Revenue Service (IRS) tax forms as individual proprietorships, partnerships, or any type of corporation, and with receipts of $1,000 or more. The ASE is conducted at the firm-level and gathers information on the firm and owner characteristics. The population represented by the survey focuses on firms with paid employees. This survey is available at an annual frequency starting in 2014.

Similar to the SBO, the ASE collects information regarding owner' and firms' characteristics for a large sample of owners. The difference is that the ASE has an annual frequency and samples only firms with paid employees. One major caveat of the ASE is that we don't have access to the microdata, therefore we use information from the tables provided by the Census Bureau to compare to our baseline estimates and explore the recent evolution in the share of firms traded.

For the table estimates provided by the Census Bureau, a business owner is defined as someone who holds more than 50% of the stake of the firm, where the firm has a positive payroll. This definition is close to our baseline definition of an entrepreneur where firms have at least one employee. Our numbers are retrieved from table `SE1600CSCBO01` where entrepreneurs are classified by the way they acquired their firm.

A.2 Robustness Exercises

A.2.1 How do Entrepreneurs Acquire Their Firms?

Owner-level. Table A.4 report how many entrepreneurs purchased their business for several alternative definitions of entrepreneurship. For example, instead of active management — as in our baseline definition — we restrict to business owners who have more than 50% of the equity of the firm, or to owners who work at least 40 hours a week in the firm. In bold we highlight our baseline definition for entrepreneurs, which implies that firm owners manage an employer firm.

Firm-level. In addition to the business owner-level results, we compute the share of firms that were acquired by their owners through a purchase. We compute the share of firms purchased in two ways: (i) if at least one entrepreneur purchased the firm; (ii) if all the firm's entrepreneurs purchased it. The results are presented in Table A.5. The purchased share computed at the firm- and owner-level are very similar. This is due to the fact that most firms have one entrepreneur, and most entrepreneurs have one firm.

Table A.4: Share of Entrepreneurs That Purchased Their Business

Sample	Purchased	N(weighted)	N
All	-	36,856,132	3,409,393
All (Respond acquisition)	16.0%	20,302,192	2,164,541
Manage	17.0%	9,503,681	1,112,254
Employment > 0	25.9%	5,507,460	1,255,134
Receipts > 0	16.9%	17,139,950	1,987,336
Payroll > 0	25.1%	6,045,634	1,338,400
Size (all) > 0	26.1%	5,344,964	1,216,319
Entrepreneur	25.5%	3,167,718	698,651
Share ≥ 50	13.5%	16,274,606	1,479,855
Share ≥ 50 and Employment> 0	23.5%	3,884,071	745,431
Share ≥ 50 and Manage	15.4%	8,064,388	827,286
Entrepreneur and Share ≥ 50	24.0%	2,458,710	469,250
Hours Worked > 40	18.0%	8,928,828	1,164,328
Hours Worked > 40 and Employment > 0	25.6%	3,505,078	802,680
Hours Worked > 40 and Manage	19.6%	5,679,652	806,923
Entrepreneur and Hours Worked > 40	26.0%	2,545,635	582,966
Entrepreneur (Weighted by Employment)	32.2%	3,167,718	698,651

Source: 2007 SBO.
Notes: Purchased refers to the percentage of entrepreneurs that acquire its firm through a purchase. Share refers to entrepreneurs' equity share. Hours Worked denotes average number of hours per week the owner spends at the firm.

As in the business owner-level results, this share is sensitive to the exclusion of firms with no employment. Definitions that consider firms with no employment tend to have lower purchasing ratios as the main input in production is probably the owner human capital, which is hard to transfer.

Franchises. We further analyze whether franchises are driving our results. Table A.6 shows that even excluding all franchises the share of entrepreneurs that purchased their firm is 16.1% and 24.2% for all firms and our baseline definition, respectively. Although is true that, within franchise owners, the share of entrepreneurs that acquired the business is very high, more than 50%, these owners represent a small group in the total number of entrepreneurs: 2.7% and 4.7% for the two definitions used.

Table A.5: Share of Firms With Owners That Purchased It

Sample	Owner-level	Firm-level	
		At least one	All
All (Respond acquisition)	16.0%	14.7%	12.0%
Manage	17.0%	16.3%	15.0%
Employment > 0	25.9%	26.8%	20.9%
Entrepreneur	25.5%	25.7%	23.2%
Entrepreneur and Hours Worked > 40	26.0%	26.1%	23.8%

Source: 2007 SBO.
Notes: Hours Worked denotes average number of hours per week the owner spend at the firm.

Table A.6: Share of Firms Purchased: Franchises

Sample	All firms	Employer firms
Baseline	17.0%	25.5%
W/o franchises	16.1%	24.2%
Franchises only	50.1%	51.8%
Share of Franchises	2.7%	4.7%

Source: 2007 SBO.

A.2.2 Firm Buyers' Previous Occupation

Alternative Computations. In the main text we documented 66% of current entrepreneurs have never been self-employed (and hence have never been entrepreneurs) prior acquiring its firm. As a robustness we check how many workers, or not self-employed, transition into entrepreneurship by acquiring its firm considering alternative definitions. In Table A.7 we compute the transition rate from worker to entrepreneur conditional on purchasing the firm for: (i) our baseline definition; (ii) when transition to being the main owner of the firm; and (iii) conditional on large firms. Our results are very similar for all these samples.

Firms' Characteristics. We also analyze whether workers tend to buy firms with certain characteristics. For example, one could argue that worker-buyers concentrate in small non-growth-oriented type of businesses, compared to firms that are acquired by previous firm owners. Table A.8 shows that there is no stark relation between firm characteristics when purchased and the share of firms purchased by workers and, if something, the share

Table A.7: Firm Buyers' Previous Occupation

Sample	Worker Before Purchasing	
	All firms	Employer firms
Baseline	62.0%	65.9%
Share> 50	61.2%	62.2%
Large Firms	66.9%	69.6%

Source: 2007 SBO.
Notes: Large Firms as those in the top quintile of the employment distribution.

is slightly larger for older and bigger firms.[33]

Table A.8: Share of Firm Buyers Who Were Workers

	Workers	Purchased
By Firm Age		
0-2	50.5%	37.0%
3-7	54.7%	14.0%
8-17	56.9%	16.0%
$\geq$ 18	60.7%	33.0%
By Firm Size		
Q1	54.2%	22.9%
Q2	54.0%	27.7%
Q3	55.3%	16.4%
Q4	56.4%	22.6%
Q5	58.7%	10.4%

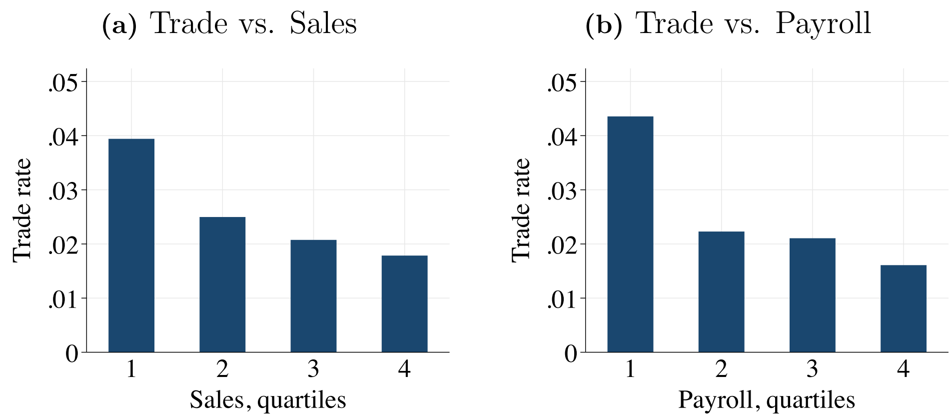
Source: 2007 SBO.
Notes: For our calculation we limit to firms purchased in the same year of the survey (2007) and employer firms as in our baseline calculations. The "Workers" column correspond to the ratio of the previously non-self employed entrepreneurs that purchased the firm over the total of firms purchased. The column "Purchased" indicates the amount of firms purchased by characteristic over all firms purchased (i.e., the distribution of purchased firms).

A.2.3 Firm Size and Trade Likelihood

For robustness, we calculate the likelihood of trade across the sales and payroll distributions. As shown in Figure A.1, firms that were in the bottom quartile of the size distribution when traded are the most likely to be traded for both definitions of firm size.

[33] The sample is restricted to 2007 such that the characteristics of the firms are approximately to the ones when purchased. For this sample, the share of firm buyers that were workers is slightly lower (less than 60%) than the one of our baseline sample.

Figure A.1: Trade Rate by Firm Size



Source: SBO.
Notes: Panels (a) and (b) use data from the 2007 SBO. The trade rate is computed using information from the firms that were sold in or after 2007. Trade rates are normalized to match the aggregate of our baseline calculations.

A.3 Additional Evidence on The Market for Firms

A.3.1 Trade share across size and age.

In Section 6, we showed that firms *when purchased* tend to be small and young, consistent with the predictions of our model. In this section, we analyze the share of entrepreneurs that purchased their firm, at any point in the past, conditional on firm observables such as size and age.

Firm Size. Table A.9 presents the share of entrepreneurs that purchased their business across the firm size distribution using three different variables of firm size: receipts, payroll, and employment. We find that the share of traded firms is even higher at the top of the size distribution. For example, in the top 0.1% of receipts, around 39% of entrepreneurs purchased their firm, considerably higher than the unconditional 25.5% share in our baseline calculations.

Firm Age. Next, we study the share of traded firms conditional on the age of the firm. Table A.10 shows that that older firms tend to have larger share of trades. This is consistent either with a higher surviving rate of purchased firms, the declining in trade share we observe in the SCF data, or just a higher probability of being purchased for being around more time. Also, this may reflect some life cycle motives since older entrepreneurs probably manage older firms. Related to this, in Appendix A.3.4 we analyze potential life cycle motives for the trade of firms.

These results suggest that traded firms, *after purchased*, tend to grow bigger and live longer than non-traded firms.

Table A.9: Firms Purchases, By Firm Size Group

Percentile	Variable	Purchased	Average
Bottom 90	Receipts	24.6%	651
	Payroll	24.6%	153
	Employment	25.2%	8
Top 10\Top 1	Receipts	34.6%	8,624
	Payroll	34.5%	1,773
	Employment	37.9%	83
Top 1\Top 0.1	Receipts	43.8%	57,753
	Payroll	40.0%	9,220
	Employment	37.9%	248
Top 0.1	Receipts	39.0%	381,869
	Payroll	35.3%	49,760
	Employment	32.3%	1,374

Source: 2007 SBO.
Notes: Results are for the baseline definition (employer firms). Average is computed using both purchased and non-purchased firms. Receipts and Payroll are in thousands ('000) of USD.

Table A.10: Share of Firms Purchased, By Firm Age

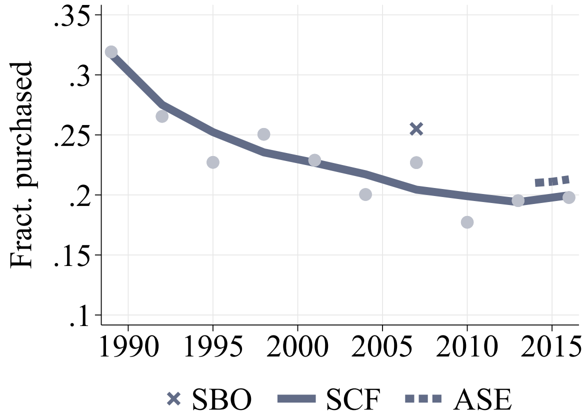
Firm Age	Owner and Manager	Entrepreneur
0-1	8.9%	17.4%
1-2	10.0%	16.3%
2-8	10.9%	16.5%
8-18	13.1%	18.5%
18-28	18.0%	24.9%
+ 28	35.5%	45.2%

Source: 2007 SBO.
Notes: The age of the firm is the age reported at the date of the survey, not when purchased.

A.3.2 Trade of Firms Across Time

Data sources. As the PUMS version of the SBO is only available for 2007, we use the SCF to document the evolution of the share of entrepreneurs that purchased their firms across different years. Table 1 shows that the SBO and SCF 2007 values are consistent. Additionally, as a robustness check, we also consider data from the ASE available for 2014 to 2016. Overall, the numbers obtained from the SCF align very well with the SBO and ASE for the years in which these surveys overlap.

Figure A.2: Fraction of Entrepreneurs that Purchased Their Business



Source: SBO, SCF and ASE.
Notes: Entrepreneurs are defined as self-employed, business owners, who actively manage their firm and the firm has at least one employee. The light-colored dots correspond to the time series SCF data points. The solid line trend was estimated using locally weighted smoothing.

Results. Figure A.2 shows that between 1989 and 2016, the fraction of entrepreneurs that acquired their firms through a purchase, which proxies for the fraction of traded firms, declined by one-third. More precisely, the fraction of entrepreneurs that purchased their business fell by 12 p.p. going from 32% in 1989 to 20% by 2016. The decreasing trend is robust to alternative definitions of entrepreneurship and changes in the sectoral composition.[34] It is worth mentioning that most of the the share of traded firms is fairly stable since 2007.

A.3.3 Ownership Structure of Private Firms

Number of Firms Owned. Using data from the SCF we document the number of businesses each entrepreneur owns and manages. Table A.11 shows that more than 80% of the entrepreneurs manage one firm at most.

Table A.11: Firms Per Entrepreneur

	# of managed businesses	
	1	≥ 2
Employer firms	83.5%	16.5%
All firms	80.2%	19.8%

DATA SOURCE: SCF 1989-2016.
Notes: Number of employer firms (baseline) and all firms per entrepreneur.

[34]These results are available upon request.

Number of Owners and Entrepreneurs. Table A.12 reports the share of firms in the 2007 SBO conditional on the number of owners and entrepreneurs. The table shows that 74% of the firms have only one entrepreneur, and 96% have at most two. If we include firms with zero employment these numbers are slightly higher (80 and 97%, respectively).

Table A.12: Share of Firms by Number of Owners and Entrepreneurs

		# of Owners			
Firms		1	2	3	≥ 4
All	Own	51.4%	39.3%	4.5%	4.8%
	+ Manage	79.8%	18.0%	1.6%	0.6%
Employer firms	Own	43.0%	42.5%	7.1%	7.4%
	+ Manage	73.7%	22.7%	2.7%	0.9%

DATA SOURCE: 2007 SBO.
Notes: Entrepreneurs are defined as (i) self-employed, (ii) business owners, who (iii) actively manage their firm. + Employment > 0 also requires that (iv) the firm has a positive number of employees. Other type of acquisition groups: acquired as a transfer, as a gift or other not specified.

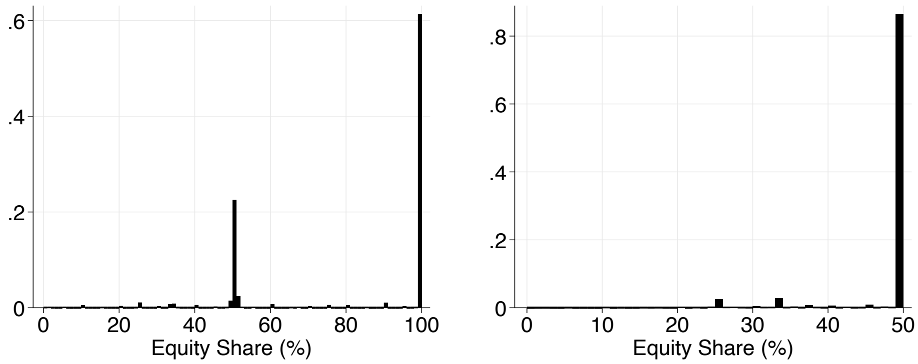
Equity Shares. Figure A.3 shows that, in our SBO sample, more than 60% of the firms have an entrepreneur that holds the 100% of the firm's equity. However, for more than 20% of the firms the entrepreneur shares around 50% of the equity with another non-manager owner. On the other hand, in firms of two entrepreneurs the most common arrangement is 50/50 equity shares. These findings are consistent with what is documented by Espino, Kozlowski, and Sanchez (2016) in other datasets. Finally, we analyze the equity share owned by entrepreneurs conditional on firm size and firm age. Figure A.4 reports that the entrepreneurs' equity shares are decreasing with both firm's size and age. Nonetheless this negative relation is relatively weak and even for the firms in the top decile of the size distribution around 75% of the firm equity is held by entrepreneurs. A similar patterns is observed across the firms' age distribution.

A.3.4 Life Cycle Motives

Another frequently cited motive for the trade of firms are motives related to the entrepreneurs' life cycle. To address this, we study the trade of firms conditional on sellers' age. Panel (a) of Figure A.5 shows that the trade rates are high for young and old entrepreneurs. This is consistent with retirement motives for older entrepreneurs and the lack of access to credit for younger entrepreneurs. However, Panel (b) of Figure A.5 shows that the share of trades is mostly concentrated among middle-aged entrepreneurs, even

Figure A.3: Equity Shares by Number of Entrepreneurs

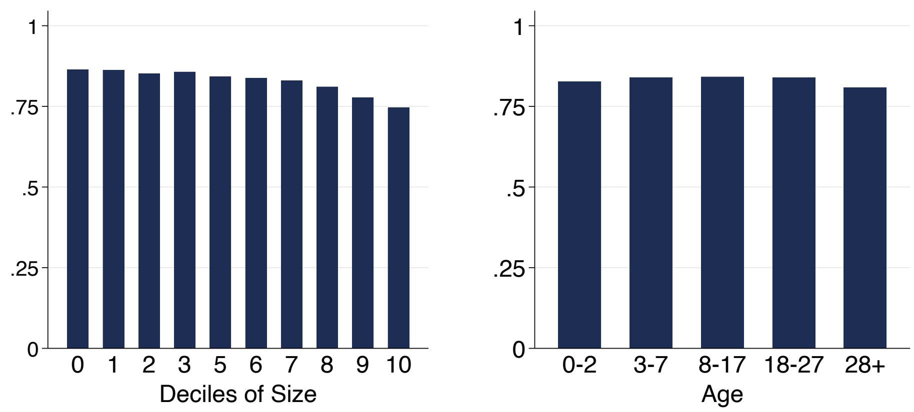
(a) One Entrepreneur **(b)** Two Entrepreneurs



Source: 2007 SBO.
Notes: Use baseline sample of employer firms.

Figure A.4: Equity Shares Across Firm Size and Age

(a) Size **(b)** Age



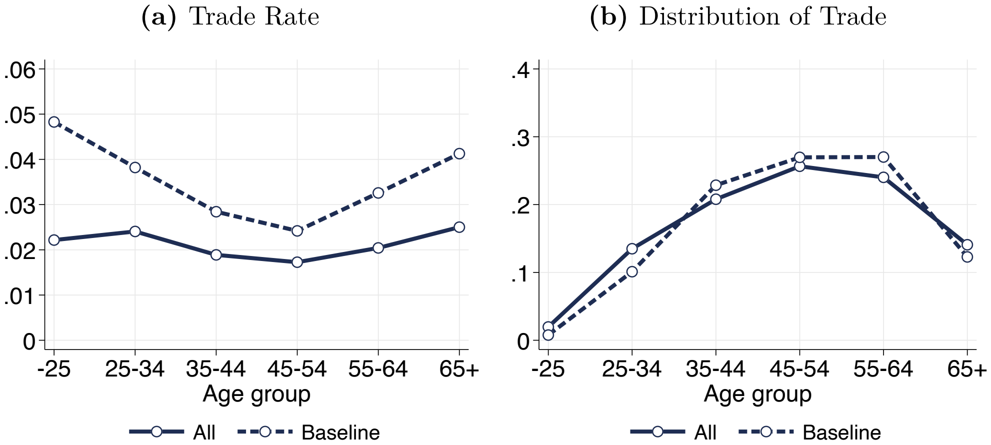
Source: 2007 SBO.
Notes: Deciles of size are constructed using the distribution of firms with positive employment. Decile 0 corresponds to firms with zero employees. Values corresponds to the average value of the sum of entrepreneurial ownership share across the firms' size and age distribution.

though these are the ones that exhibit the lowest trade rates. This result reflects the fact that the age distribution of entrepreneurs has an inverted U-shape. Thus, even though old entrepreneurs' selling rate is relatively high, the fraction of total trades that could be related to retirement, as proxied by share of sells done by entrepreneurs in the 65+ category, is just around 10%.

A.4 Firms' Trade Rate

We indirectly infer the annual trade rate by combining firm dynamics moments, such as the entry and exit rate, and the stock of purchased firms with firms' flow equations. Define the mass of all firms at t as y_t and the stock of firms purchased at t as x_t. Then,

Figure A.5: Trade of Firms by Sellers' Age Group



Source: 2007 SBO.
Notes: The trade rates in Panel (a) are normalized to match the total trade rate of 2 and 3%.

these variables follow the laws of motion

$$y_{t+1} = y_t \left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right]$$
$$x_{t+1} = x_t \left(1 - \pi^x_{exit,t}\right) + \left[y_{t+1} - x_t \left(1 - \pi^x_{exit,t}\right)\right] \pi_{trade,t_+}$$

where $\pi_{entry,t}$ and $\pi_{exit,t}$ are the annual entry rate and exit rate, respectively, and π_{trade,t_+} is the annual rate of firm trade we want to estimate. Combing the flow equations, we have that the ratio of firms traded evolves as

$$\left(\frac{x_{t+1}}{y_{t+1}}\right) = \left(\frac{x_t}{y_t}\right) \left\{\frac{1 - \pi^x_{exit,t} + \frac{y_t}{x_t}\left[1 - \pi^y_{exit,t} + \pi_{entry,t}\right] \pi_{trade,t_+} - \left(1 - \pi^x_{exit,t}\right) \pi_{trade,t_+}}{1 - \pi^y_{exit,t} + \pi_{entry,t}}\right\}$$

if the exit rate for traded and non-traded firms is equated ($\pi_{exit,t} = \pi^y_{exit,t} = \pi^x_{exit,t}$), and the entry and exit rate coincide ($\pi_{e,t} = \pi_{exit,t} = \pi_{entry,t}$), then the we can calculate the steady state annual trade rate π_{trade} using the observed exit rate of firms π_e and share of traded firms $\frac{x}{y}$ using the following equation

$$\pi_{trade} = \frac{\pi_e}{\left(\frac{x}{y}\right)^{-1} - 1 + \pi_e}.$$

B Orbis Data Appendix

This appendix describes the Orbis database. It also presents our algorithm to identify firms' transactions using Orbis' ownership files, and presents some additional results.

B.1 The Orbis Database

To document post-trade firm dynamics, we use the historical product of Orbis, an extensive firm-level database covering millions of companies worldwide. This database is compiled by Moody's Bureau van Dijk (BvD), which aggregates data from various sources, such as national business registries, and harmonizes it into a globally comparable format.

Industry Files From Orbis, we use the industry files reporting annual balance sheets and income statements for private and publicly traded firms. The industry files contain information starting from the early 1990s to 2019. We compute firm-level output, capital, and ARPK measures using these files. This data also includes information about firms' use of inputs, country and industry identifiers, and the year they were founded.

Ownership Files In addition, we use Orbis' ownership files to identify trades in the market for firms. From 2007 onward, this database reports annual snapshots with the list of owners for a large number of firms. The data reports owners' names, with unique identifiers we can track over time, and the owners' equity shares in the respective firms. As described below, we identify trades using changes in owners' names through a string similarity algorithm that excludes spurious discrepancies or changes that could be related to inheritances and family-related transfers.

Sample Selection We focus on a sample of European private firms, which are the ones best covered by Orbis. For our baseline results, we study eleven high-income European countries which are the most comparable to the US: Austria, Belgium, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, and the United Kingdom. Below, we also present results for a group of ten middle-income European countries, which include: Bulgaria, Croatia, Czechia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, and Slovenia. As documented in Kochen (2023), given the high cross-country correlation between income and finance, the high- and middle-income labels coincide with labels for developed and less financially developed countries. Our analysis focuses on the firm-year observations from 2006 to 2019 with available capital, output, and ownership data. Further, we restrict to the firms we observe for at least five years. Table B.1 presents descriptive statistics of our primary sample. The table shows that while the firms in Orbis with available ownership data are somewhat larger, they are similar in terms of firms' age and capital growth to the complete sample.

Variables' Definitions We follow Kochen (2023) for our definitions of the main variables using the balance sheet and income statements from Orbis. We measure firms' capital as equity plus net financial debt: $k_{it} = e_{it} + b_{it}$, where using Orbis acronyms, we measure equity as $e_{it} = \texttt{toas}_{it-1} - \texttt{culi}_{it-1} - \texttt{ncli}_{it-1}$ and net financial debt as $b_{it} = \texttt{loan}_{it-1} + \texttt{ltdb}_{it-1} - \texttt{cash}_{it-1}$. The variable `toas` denotes total assets, `culi` is current liabilities, `ncli` is non-current liabilities, `loan` is short-term financial debt (payable within a year), `ltdb` is long-term financial debt, and `cash` denotes the firm's cash and cash equivalents. Balance sheet variables in the data are reported at the end of each year. Hence, to be consistent with the model, we use the one-period lag to measure the beginning of the period variable. We measure firms' output using value-added, defined as revenue minus materials: $y_{it} = \texttt{opre}_{it} - \texttt{mate}_{it}$. Labor costs are $wl_{it} = \texttt{staf}_{it}$. We measure firms' profits as the sum of profits plus all extraordinary revenues minus extraordinary expenses $\pi_{it} = \texttt{plat}_{it} + \texttt{extr}_{it}$.[35]

Table B.1: Orbis Database Descriptive Statistics

	High-Income		Middle-Income	
	Mean	SD	Mean	SD
All Firms				
Age	16.0	13.0	12.5	8.2
Output	2.9	54.4	1.5	14.5
$\Delta \log(k)$	0.043	0.58	0.073	0.61
Obs.	16,247,768		4,252,636	
Firms w/ Ownership				
Age	17.1	13.7	12.2	7.7
Output	4.4	72.1	1.5	12.4
$\Delta \log(k)$	0.046	0.59	0.072	0.59
Obs.	8,548,886		2,203,131	

Notes: Descriptive statistics for our sample of firms between 2006-2019, with available output and capital, and observed for at least five years. All Firms are the observations in the Industry Files satisfying these criteria. Firms w/ Ownership are the observations that, in addition, have available data in the Ownership Files. Age is in years, Output is in million 2015 USD at constant exchange rates, and $\Delta \log(k)$ is capital's one-year growth rate measured in log changes.

[35] The definition of capital we use for Orbis is similar to the one we used for the KFS. Appendix A.2 in Kochen (2023) shows that tangible assets and inventories account for the bulk of the balance sheet categories in k, which also includes intangible assets and a fourth category. Our definition of output y in Orbis subtracts for materials, a variable we don't have available in the other data sets. Our results are almost identical if we define output using only revenue, as in the SBO.

B.2 Algorithm to Identify Trades in the Market for Firms

Using the Orbis database, we use the following methodology to identify trades in the market for firms. First, to be consistent with our model, we focus on a sample of firms where one owner holds at least 50% of the equity. We then identify firms' trades by tracking these majority owners' identities over time. After selecting all the years with a change in the name of the top owner, we compute four string similarity metrics for all the old and new owners' pairs: Jaro-Winkler distance; Levenshtein distance, normalized by the largest string length among the two names; Soundex; and Token Soundex measures. All these metrics lie in the [0,1] interval. After computing these measures, we exclude all the pairs that satisfy at least one of the following conditions:

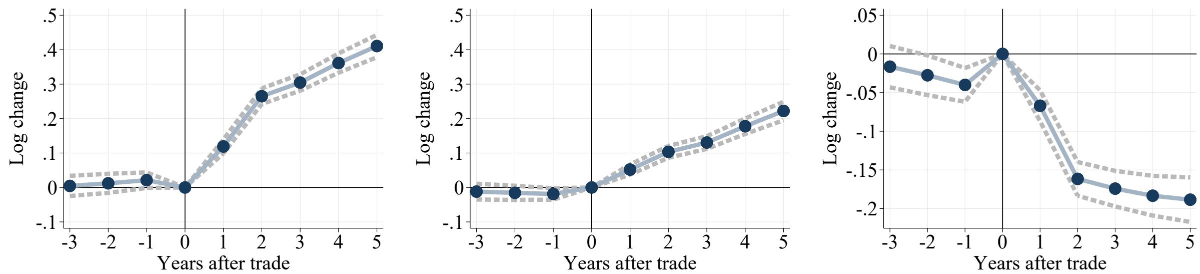
1. The pair is in the top 25 percentile of similarity according to Jaro-Winkler.
2. The pair is in the top 25 percentile of similarity according to Levenshtein.
3. Soundex is equal to 1.
4. Token Soundex is equal to 1.

Conditions 1-4 exclude changes that might be spurious due to mistakes or slight name changes. In addition, this algorithm excludes changes related to inheritances or family transfers as it would identify, for example, the pair of names that share the same last name. Finally, to exclude temporary changes, we also focus on the firms we observe being traded only once in our sample period. Our main results are robust to using alternative similarity metrics or varying the percentile thresholds in conditions 1. and 2.

B.3 Additional Results

Figure B.1: Capital, Output, and ARPK Dynamics After Trade, Extended Window

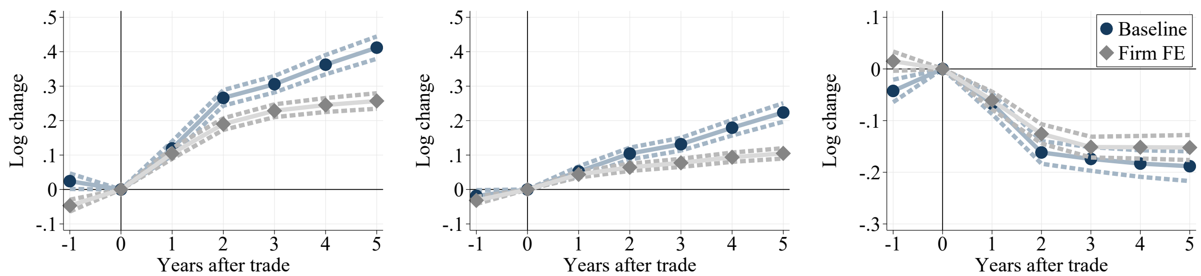
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1) considering an extended window from -3 to 5 years after trade. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.2: Capital, Output, and ARPK Dynamics After Trade, Firm Fixed Effects

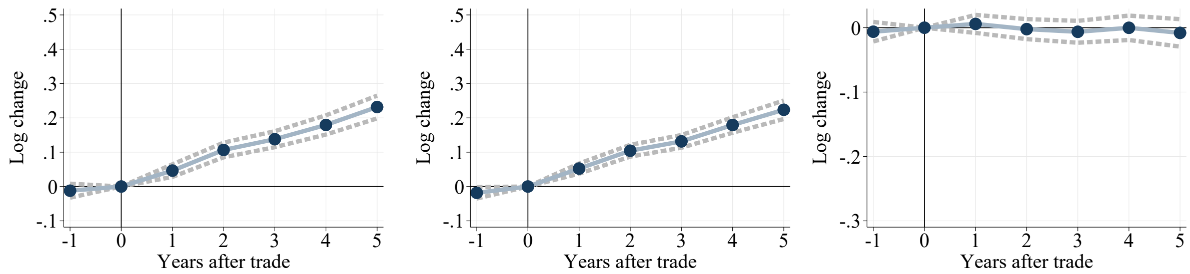
(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$



Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1) considering firm fixed effects. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.3: Labor, Output and ARPL Dynamics After Trade

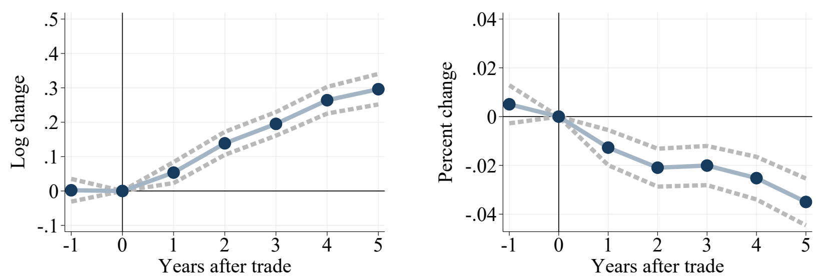
(a) Labor, $\log(wl)$ **(b)** Output, $\log(y)$ **(c)** ARPL, $\log(y/wl)$



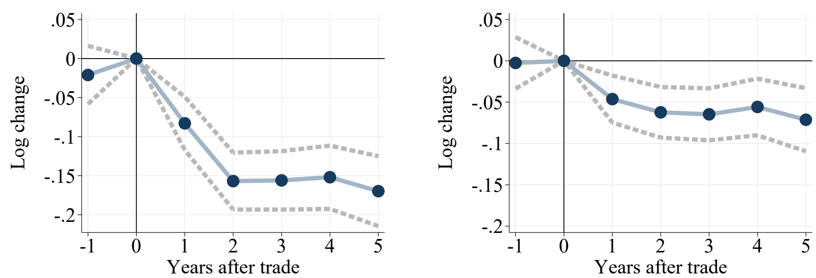
Source: Orbis Historical.
Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.4: Firms' Profits After Trade in the Orbis Data

(a) Profits, $\log(\pi)$ **(b)** Neg. Profits, $\mathbb{1}\{\pi \leq 0\}$

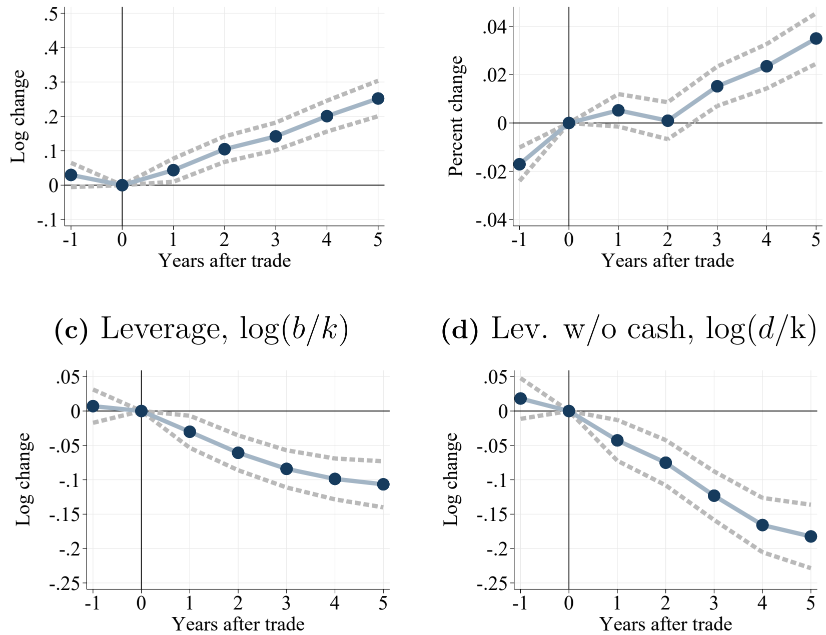


(c) Profitability, $\log(\pi/k)$ **(d)** ROA, $\log(\pi/\text{assets})$



Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

Figure B.5: Firms' Debt After Trade in the Orbis Data

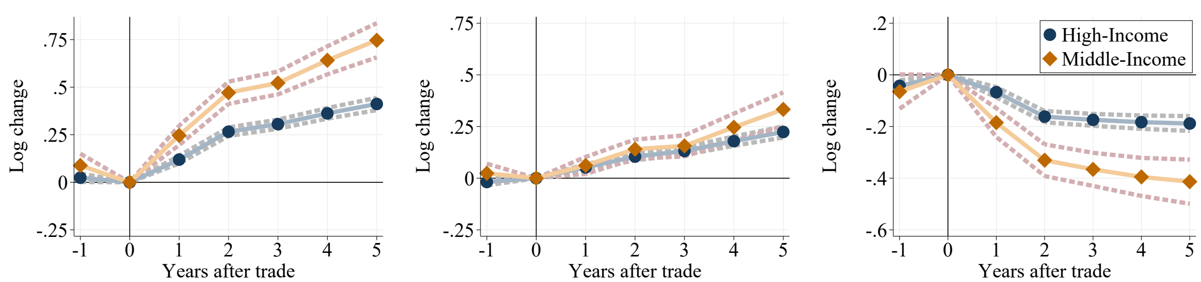


(a) Net Debt, $\log(b)$ **(b)** Neg. Net Debt, $\mathbb{1}\{b \leq 0\}$

(c) Leverage, $\log(b/k)$ **(d)** Lev. w/o cash, $\log(d/\mathrm{k})$

Notes: Estimated coefficients $\hat{\beta}_h$ from (1). The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors. The variable b is net financial debt, $\texttt{loan} + \texttt{ltdb} - \texttt{cash}$, while d is financial debt, $\texttt{loan} + \texttt{ltdb}$.

Figure B.6: Firm Dynamics After Trade in High- and Middle-Income Countries



(a) Capital, $\log(k)$ **(b)** Output, $\log(y)$ **(c)** ARPK, $\log(y/k)$

Notes: Estimated coefficients $\hat{\beta}_h$ from (1), separately for high- and middle-income European countries. The dashed lines correspond to 99% confidence intervals considering firm-level clustered standard errors.

C Model Appendix

This appendix includes additional derivations and results from our model. It also presents the recursive formulation of firm owners and workers, the competitive equilibrium definition, and a detailed description of our computation solution.

C.1 Additional Derivations

To simplify the notation, we turn to the recursive notation in steady-state.

C.1.1 Private Firms' Optimality Conditions

The solution of entrepreneurs' profit maximization problem, stated in (2), is characterized by the input demand functions

$$k(a,z) = \min\left\{k^*(z),\ \lambda a\right\}, \qquad l(a,z) = \left[\frac{z\nu}{w}\right]^{\frac{1}{1-\nu}} k(a,z)^{\frac{\theta}{1-\nu}},$$

where k^* is the unconstrained optimal level of capital given by

$$k^*(z) = z^{\frac{1}{1-\theta-\nu}} \left[\frac{\theta}{R}\right]^{\frac{1-\nu}{1-\theta-\nu}} \left[\frac{\nu}{w}\right]^{\frac{\nu}{1-\theta-\nu}}$$

which is only a function of the quality of the entrepreneurial project z.

C.1.2 Public Firm's Optimality Conditions.

The FOCs of the public firm's profit maximization problem are

$$\eta\frac{Y_p}{K_p} = R, \qquad (1-\eta)\frac{Y_p}{L_p} = w$$

which imply a relation between the public firm's capital to output and prices.

C.2 Recursive Formulation

We now present the recursive problem of firm owners and workers. First, we describe the value functions at the beginning of the market for firms (the DM subperiod), which we denote by V. Second, we present the value functions at the production stage (the CM subperiod), which we denote by W.

C.2.1 Value at the Market for Firms (DM)

Firm owners have four potential outcomes upon entering the market for firms: (1) don't trade, (2) buy another firm, (3) sell their firm to another owner, and (4) sell their firm to a worker. The no-trade case could arise because the owner did not match with a counterpart or because there was a match, but it did not end with a trade.

The value of a firm owner with states (a_{it}, z_{it}) at the beginning of DM is equal to

$$V^o(a_{it}, z_{it}) = \mathbb{E}_{\kappa_{it}} \Bigg[\underbrace{\Pr^o[\text{no trade} \mid a_{it}, z_{it}, \kappa_{it}] \; W^o(a_{it}, z_{it})}_{\text{no trade}}$$
$$+ \underbrace{\alpha_o \int \int_{z_{it} < z_{jt}, \overline{p}_{it} > \underline{p}_{jt}} W^o(a_{it} - p, z_{jt}) \, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt}) \, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}}$$
$$+ \underbrace{\alpha_o \int_{z_{it} > z_{jt}, \underline{p}_{it} < \overline{p}_{jt}} [W^w(a_{it} + p, \underline{\varepsilon}) + T_{it}(p)] \, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt})}_{\text{sell to a firm owner}}$$
$$+ \underbrace{\alpha_w \int_{\underline{p}_{it} < \overline{p}_{jt}} [W^w(a_{it} + p, \underline{\varepsilon}) + T_{it}(p)] \, \mathrm{d}N^w_{dm}(a_{jt}, \varepsilon_{jt})}_{\text{sell to a worker}} \Bigg], \tag{10}$$

where α_o and α_w are exogenous matching probabilities conditional on each match type.[36] These parameters, in $[0, 1]$, govern the degree of search frictions in the market for firms. N^o_{dm} and N^w_{dm} are cumulative distributions for firm owners and workers at the beginning of DM, which satisfy that $\int \mathrm{d}N^o_{dm} + \int \mathrm{d}N^w_{dm} = 1$.

As mentioned in Section 3.2, for the case of owner-owner meetings, who buys and sells depends on the relative firm qualities. Hence, an owner with firm quality z_{it} might buy if it is matched with another owner with a firm of higher quality ($z_{it} < z_{jt}$), as denoted in the integral in the second line of (10). On the contrary, the owner might sell if it is matched with another owner with a firm of lower quality ($z_{it} > z_{jt}$) as denoted in the integral of the third line. Note that the integrals for the buying and selling cases consider only the meetings that result in a trade, which occurs when the seller's minimum price is lower than the buyer's maximum price, as stated in (8).

Workers only have two potential outcomes: (1) don't trade, or (2) buy an existing firm. Hence, the value of a worker with states $(a_{it}, \varepsilon_{it})$ at the beginning of DM is given by

$$V^w(a_{it}, \varepsilon_{it}) = \underbrace{\Pr^w[\text{no trade} \mid a_{it}, \varepsilon_{it}] \; W^w(a_{it}, \varepsilon_{it})}_{\text{no trade}}$$
$$+ \underbrace{\alpha_w \int \int_{\overline{p}_{it} > \underline{p}_{jt}} W^o(a_{it} - p, z_{jt}) \, \mathrm{d}N^o_{dm}(a_{jt}, z_{jt}) \, \mathrm{d}\Psi(\kappa_{jt})}_{\text{buy}}. \tag{11}$$

[36] In more detail, the probabilities of the bilateral meetings in (10) can be derived as follows. First, note that there is a mass $\int \mathrm{d}N^o_{dm}$ of owners at the beginning of DM. This implies that two owners are matched with probability $\int \mathrm{d}N^o_{dm}$. Due to the search friction, conditional on the match, these owners meet with probability α_o. Thus, the probability of an owner-owner meeting is equal to $\alpha_o \int \mathrm{d}N^o_{dm}$. Similarly, the probability that the owner matches with a worker is equal to $\int \mathrm{d}N^w_{dm} = 1 - \int \mathrm{d}N^o_{dm}$, and conditional on the match they meet with probability α_w. Hence, the probability of an owner-worker meeting is equal to $\alpha_w \int \mathrm{d}N^w_{dm}$. Finally, note that the no-trade probability $\Pr^o[\text{ no trade} \mid a, z\ ]$ sums up the probability of no meetings plus the probability of meetings that do not result in a trade as $\underline{p} < \overline{p}$ is not satisfied.

C.2.2 Value at the Production Stage (CM)

As previously described, firm owners face an occupational choice at the beginning of the production stage. They have to decide whether to operate the firm and be entrepreneurs or shut down and go to the labor market with labor productivity $\underline{\varepsilon}$. Given these assumptions, the value of firm owners at the beginning of CM is

$$W^o(a_{it}, z_{it}) = \max_e \{W^e(a_{it}, z_{it}), W^w(a_{it}, \underline{\varepsilon})\} \tag{12}$$

where e denotes the owners' occupational choice.

The value function of entrepreneurs is given by

$$\begin{aligned} W^e\left(a_{it}, z_{it}\right) = \max_{a_{it+1}, c_{it}} \quad & u\left(c_{it}\right) \;+\; \beta\left\{\gamma V^o\left(a_{it+1}, z_{it}\right) + (1-\gamma)\mathbb{E}_{z_{it+1}}\left[V^o\left(a_{it+1}, z_{it+1}\right)\right]\right\} \\ \text{s.t.} \quad & c_{it} = \pi(a_{it}, z_{it}) + (1+r)a_{it} - a_{it+1} \\ & c_{it} \geq 0,\; a_{it+1} \geq 0 \end{aligned} \tag{13}$$

and the value function of workers by

$$\begin{aligned} W^w\left(a_{it}, \varepsilon_{it}\right) = \max_{a_{it+1}, c_{it}} \quad & u\left(c_{it}\right) + \beta\left\{\zeta\mathbb{E}_{\varepsilon_{it+1}|\varepsilon_{it}}\left[V^w\left(a_{it+1}, \varepsilon_{it+1}\right)\right] + (1-\zeta)\mathbb{E}_{z_{it+1}}\left[V^o\left(a_{it+1}, z_{it+1}\right)\right]\right\} \\ \text{s.t.} \quad & c_{it} = \varepsilon_{it} w + (1+r)a_{it} - a_{it+1} \\ & c_{it} \geq 0,\; a_{it+1} \geq 0 \end{aligned} \tag{14}$$

where $(1-\zeta)$ is the probability of the exogenous startup shock through which a worker can become a firm owner.[37]

C.3 Competitive Stationary Equilibrium

A *competitive stationary equilibrium* in this economy consists of: (i) aggregate prices $\{r, R, w\}$; (ii) terms of trade in the market for firms given by the price functions of seller j and buyer-owner i meetings $\{p\left(\mathbf{s}^o_{it}, \mathbf{s}^o_{jt}, \kappa_{jt}\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^o_{it}, z_{jt}\right)\}$, and the price functions of seller j and buyer-worker i meetings $\{p\left(\mathbf{s}^w_i, \mathbf{s}^o_j, \kappa_j\right), \underline{p}\left(\mathbf{s}^o_j, \kappa_j\right), \bar{p}\left(\mathbf{s}^w_{it}, z_{jt}\right)\}$; (iii) firm owners' occupational choice decisions $e(a_{it}, z_{it})$; (iv) consumption and savings decisions for entrepreneurs $\{c(a_{it}, z_{it}), a'(a_{it}, z_{it})\}$ and for workers $\{c(a_{it}, \varepsilon_{it}), a'(a_{it}, \varepsilon_{it})\}$; (v) capital and labor demand functions for private and public firms, $\{k(a_{it}, z_{it}), l(a_{it}, z_{it}), K_{pt}, L_{pt}\}$; and (vi) measures of agents over occupations and idiosyncratic states at DM and CM subperiods characterized by $\{N^o_{dm}(a_{it}, z_{it}), N^w_{dm}(a_{it}, \varepsilon_{it})\}$ and $\{N^e_{cm}(a_{it}, z_{it}), N^w_{cm}(a_{it}, \varepsilon_{it})\}$, respectively, such that:

[37] In (13) and (14) we omit the profits of the public firm and the financial intermediary (Π^p and Π^f terms) in the households' budget constraints as both terms are equal to zero, in equilibrium.

1. In DM, the terms of trade in bilateral meetings solve the Nash bargaining problem.
2. In CM, given prices, households, private, and public firms solve their corresponding optimization problems.
3. Goods market clears, period by period:

$$Y_t = C_t + K_{t+1} - (1-\delta)K_t \tag{15}$$

where

$$\begin{aligned} Y_t &\equiv Y_{pt} + \int z_{it} k(a_{it}, z_{it})^{\theta} l(a_{it}, z_{it})^{\nu}\ \mathrm{d}N^e_{cm}(a_{it}, z_{it}) \\ C_t &\equiv \int c(a_{it}, z_{it})\ \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int c(a, \varepsilon) \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) \\ K_t &\equiv K_{pt} + \int k(a_{it}, z_{it})\ \mathrm{d}N^e_{cm}(a_{it}, z_{it}). \end{aligned}$$

4. Labor market clears, period by period:

$$L_{pt} + \int l(a_{it}, z_{it})\ \mathrm{d}N^e_{cm}(a_{it}, z_{it}) = \int \varepsilon_{it}\ \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}). \tag{16}$$

5. The budget constraint of the financial intermediary, specified in (3), is satisfied period by period.
6. The measures over types and states satisfy

$$\begin{aligned} \int \mathrm{d}N^o_{dm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{dm}(a_{it}, \varepsilon_{it}) = 1 \\ \int \mathrm{d}N^e_{cm}(a_{it}, z_{it}) + \int \mathrm{d}N^w_{cm}(a_{it}, \varepsilon_{it}) = 1 \end{aligned}$$

and are consistent with a recursive equilibrium mapping dictated by prices and trades in the market for firms, households' optimal choices, and the stochastic processes for firms' qualities, workers' labor efficiencies, and sellers' preferences shocks. The stationary equilibrium implies that fixed distribution over time (fixed point).

C.4 Computational Solution

To solve the model we use projection methods to approximate the value functions $\{V^o, W^o, V^w, W^w\}$. Thus, we need to solve for coefficients $\{g^o_V, g^o_W, g^w_V, g^w_W\}$ such that, at the grid points, satisfy: $V^o(a,z) = \Phi^z(a,z)g^o_V$, $W^o(a,z) = \Phi^z(a,z)g^o_W$, $V^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g^w_V$, and $W^w(a,\varepsilon) = \Phi^\varepsilon(a,\varepsilon)g^w_W$. Note that the FOCs of the public firm give us a relation between K_p/Y_p, w and r. Both K_p and L_p are determined as residuals from the market clearing conditions of capital and labor, thus we can obtain w as a function of r.

This considerably simplifies the solution method of our baseline model as we only need to solve for one equilibrium price: r.

C.4.1 Algorithm

The equilibrium objects we need to solve for are

$$\left\{\underline{p}, \overline{p}, p, g_V^o, g_W^o, g_V^w, g_W^w, n_{dm}^o, n_{dm}^w, n_{cm}^o, n_{cm}^e, P_{dm}^o, P_{dm}^w, P_{cm}^o, P_{cm}^w, \beta\right\}$$

where $\underline{p}$ are sellers' minimum prices, $\overline{p}$ are buyers' maximum prices, p are the Nash bargaining prices, n are the probability densities across states, and P are the transition probability matrices (TPMs).[38] We solve for these objects using the following algorithm:

Iteration on prices

0. Propose an initial guess for r.
1. Given r, solve the model (in partial equilibrium).

   Iteration on distributions

   1.0. Propose an initial guess for $\{n_{dm}^o, n_{dm}^w\}$.

   1.1. Given $\{n_{dm}^o, n_{dm}^w\}$, solve for $\{g_W^o, g_W^w\}$.

   Iteration on value functions

   1.1.0. Propose an initial guess for $\{g_W^o, g_W^w\}$.

   1.1.1. Solve for the prices in the market for firms $\{\underline{p}, \overline{p}, p\}$.

   1.1.2. Solve the DM problem: get $\{g_V^o, g_V^w\}$.

   1.1.3. Solve the CM problem: obtain e, a' and P_{cm}.

   1.1.4. Update $\{g_W^o, g_W^w\}$.

   1.1.5. Iterate $\{g_W^o, g_W^w\}$ until convergence.

   1.2. Update $\{n_{dm}^o, n_{dm}^w\}$.

   1.3. Iterate $\{n_{dm}^o, n_{dm}^w\}$ until convergence.
2. Update r such that the capital market clears.
3. Return to 1. until r converges.

C.4.2 Solving for Prices in the Market for Firms

First, for each potential seller (a, z, κ), we solve for the sellers' minimum price by finding $\underline{p}(a, z, \kappa)$ that implies a sellers surplus, defined in (4) and (5), equal to zero. Using (7), which defines the preference shock utility transfer, the seller's surplus is equal to zero if

$$W^w(a + \kappa\underline{p}, \underline{\varepsilon}) = W^o(a, z)$$

which implicitly defines $\underline{p}(a, z, \kappa)$.

[38] Where $\int n^o(a, z)\mathrm{d}a\mathrm{d}z = s^o$ and $\int n^w(a, \varepsilon)\mathrm{d}a\mathrm{d}\varepsilon = (1 - s^o)$.

Second, for each potential firm quality z_j, we solve for buyers' maximum price $\overline{p}(\mathbf{s}_i, z_j)$, where $\mathbf{s}_i \in \{\mathbf{s}_i^o, \mathbf{s}_i^w\}$ depending on whether the buyer is a firm owner or a worker. Note that buyers' maximum price is only a function of the seller's firm quality and does not depend on the seller's assets or the preference shock. We compute the buyer's maximum price by solving for $\overline{p}$ that sets the buyer's surplus, defined in (4) and (5), to zero. For the case of current business owners with states $\mathbf{s}_i^o = (a_i, z_i)$, note that they will never buy a lower quality firm $z_j < z_i$. For those cases, we set the buyers' maximum price equal to zero.

Having computed the sellers' minimum prices, $\underline{p}$, and the buyers' maximum prices $\overline{p}$, we can identify the matches with positive gains from trade using the sufficient condition presented in (8). Then, for each potential match of a seller, with states $(\mathbf{s}_j^o, \kappa_j)$, and a buyer, with states $\mathbf{s}_i$, such that there are positive gains from trade, given by $\underline{p}(\mathbf{s}_j^o, \kappa_j) < \overline{p}(\mathbf{s}_i, z_j)$, we approximate the Nash bargaining price, defined in (9), as

$$p(\mathbf{s}_{it}, \mathbf{s}_{jt}^o, \kappa_{jt}) \approx \chi \underline{p}(\mathbf{s}_j^o, \kappa_j) + (1 - \chi)\overline{p}(\mathbf{s}_i, z_j) \tag{17}$$

where $\mathbf{s}_i \in \{\mathbf{s}_i^o, \mathbf{s}_i^w\}$ and χ is the buyers' bargaining power. In our numerical simulations, we found that computing the price using (17) is an extremely accurate approximation to the Nash bargaining price obtained from solving the maximization problem (9) while delivering improvements in computational time of several orders of magnitude.

C.4.3 Solving for g_V^o and g_V^w

Given $\{\underline{p}, \overline{p}, p, n_{dm}^o, n_{dm}^w, g_W^o, g_W^w\}$, we can compute the value at DM for firm owners and workers. Then we can solve for g_V^o and g_V^w by inverting the basis functions Φ^z and Φ^ε.

C.4.4 Solving for a', g_W^o and g_W^w

Having solved for the coefficients g_V^o and g_V^w we can solve the households' problems in the production stage (CM). Given r and w, both entrepreneurs and workers problems are a single variable optimization problem in a', which we can solve using golden search.

To obtain g_W^o and g_W^w we use value function iteration. First, by substituting the corresponding optimal policies we obtain two linear systems of equations on g_W^o and g_W^w. Then, we can solve for the coefficients by just inverting the basis functions. For stability reasons we make the update of g_W^o and g_W^w with some dampening.

C.4.5 Transitions and Stationary Distribution

Define the densities across states in DM and CM subperiods as

$$n_{dm} = \begin{bmatrix} n_{dm}^o \\ n_{dm}^w \end{bmatrix} \text{ and } n_{cm} = \begin{bmatrix} n_{cm}^o \\ n_{cm}^w \end{bmatrix}$$

where n^o_{dm} and n^o_{cm} are vectors of size N_o and n^w_{dm} and n^w_{cm} are vectors of size N_w. N_o and N_w are the basis functions grid sizes denoting the number of (a, z) and (a, ε) combinations, respectively. Here $\sum_i n_{dm} = 1$, thus, $\sum_i n^o_{dm} = s^o_{dm}$ and $\sum_i n^w_{dm} = (1 - s^o_{dm})$.

Then, the TPMs between DM and CM and CM and DM$_{+1}$ solve

$$(n_{cm})^\intercal = (n_{dm})^\intercal P_{dm}, \qquad (n'_{dm})^\intercal = (n_{cm})^\intercal P_{cm}$$

where $(.)^\intercal$ denotes the transpose operator.

We can divide the TPM in blocks differentiating between the two type of agents:

$$P_{dm} = \begin{bmatrix} P^{oo}_{dm} & P^{ow}_{dm} \\ P^{wo}_{dm} & P^{ww}_{dm} \end{bmatrix} \text{ and } P_{cm} = \begin{bmatrix} P^{oo}_{cm} & P^{ow}_{cm} \\ P^{wo}_{cm} & P^{ww}_{cm} \end{bmatrix}$$

where P^{oo}_{dm} captures the transitions of firms' owners that bought another firm or didn't trade, P^{ow}_{dm} is for owners that sold their firm, P^{wo}_{dm} for workers who bought a firm and P^{ww}_{dm} for workers who didn't trade. Regarding CM TPMs, P^{oo}_{cm} is for business owners who operated the firm, P^{ow}_{cm} for owners who didn't operate and went to the labor market, P^{wo}_{cm} for workers who received the $(1 - \zeta)$ shock, P^{ww}_{cm} for workers that didn't. Note that besides changes in the exogenous shocks, asset holdings also change due to payments in the market for firms and due to savings in CM.

Stationarity requires that

$$n^\intercal_{dm} = n^\intercal_{dm} P_{dm} P_{cm}$$

or

$$[I - (P_{dm} P_{cm})^\intercal]\, n_{dm} = 0$$

which implies that we can solve for n_{dm} by computing the eigenvector of $(P_{dm} P_{cm})^\intercal$ associated with the unit eigenvalue, normalized such that $\sum_i n_{dm}(i) = 1$.

C.5 Post-Trade Firm Dynamics: Analytical Results

In this appendix, we derive analytical predictions for the post-trade dynamics of the six variables analyzed in Section 2.2, when firms' trade is driven by: (i) financial frictions and heterogeneity in owners' wealth, (ii) heterogeneity in owners' managerial skills, and (iii) heterogeneity in owners' span-of-control. First, we demonstrate how profits change with lower financial frictions, improved managerial skills, and increased span of control. Then, we present our main proposition, which characterizes the evolution of output, capital, ARPK, ARPL, profitability, and leverage under these different motives.

To study these alternative motives, we extend the private firms' problem in (2) by assuming that firms use a production technology that combines capital, labor, and managerial skills m such that output is $y = zm^{\epsilon}\left(k^{\alpha}l^{1-\alpha}\right)^{\nu}$ where $\nu \in (0,1)$ captures the production scale over labor and capital, and $\epsilon \geq 0$ determines the importance of managerial skills in output. Firms hire labor at wage w and rent capital at a gross rate R. The amount of capital firms can use for production might be constrained by the financing friction $k \leq \lambda a$. We let managerial skills influence output and the firm's costs. In particular, we assume the firm faces a fixed cost $\phi(m) \geq 0$, which is decreasing in managerial skills m. Without loss of generality, we assume the firm can have negative profits. In this setup, the firm profit function is then given by

$$\pi(z,m,a) = \max_{k\leq\lambda a,l} zm^{\epsilon}\left(k^{\alpha}l^{1-\alpha}\right)^{\nu} - wl - Rk - \phi(m). \tag{18}$$

The optimality conditions are

$$[k]: \quad zm^{\epsilon}\nu\alpha\left(k^{\alpha}l^{1-\alpha}\right)^{\nu-1}k^{\alpha-1}l^{1-\alpha} = R+\mu \tag{19}$$

$$[l]: \quad zm^{\epsilon}\nu(1-\alpha)\left(k^{\alpha}l^{1-\alpha}\right)^{\nu-1}k^{\alpha}l^{-\alpha} = w \tag{20}$$

where μ is the Lagrange multiplier on the capital collateral constraint. From the FOC, we get the capital and labor demand functions

$$k = \left[\frac{zm^{\epsilon}\nu\alpha l^{(1-\alpha)\nu}}{(R+\mu)}\right]^{\frac{1}{1-\alpha\nu}}$$

$$l = \left[\frac{zm^{\epsilon}\nu(1-\alpha)k^{\alpha\nu}}{w}\right]^{\frac{1}{1-(1-\alpha)\nu}}.$$

In addition, firms' ARPK and ARPL are

$$\frac{y}{k} = \frac{(R+\mu)}{\nu\alpha} \tag{21}$$

$$\frac{y}{wl} = \frac{1}{\nu(1-\alpha)}. \tag{22}$$

Due to financial frictions, the ARPK is weakly higher than the marginal cost of capital R.

Assumption 1. *We restrict the prices and parameters to be such that:*

$$zm^{\epsilon}\nu\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)}\left(\frac{\alpha}{R}\right)^{\alpha} > 1.$$

We impose this restriction to simplify the analysis of the span-of-control case in Lemma 1 and Proposition 1.[39]

[39]This restriction implies that the $l^{1-\alpha}k^{\alpha}$ is greater than one if the firms are unconstrained, to rule out that the greater span-of-control reduces output.

Lemma 1. *Firm's profits* $\pi(z,m,a)$ *are:*

(i) weakly increasing in owner's wealth a*,*

(ii) strictly increasing in managerial abilities m *if the firm is unconstrained,*

(iii) and increasing in the span-of-control ν *assuming the firm is unconstrained and Assumption 1 holds.*

Proof. (Lemma 1) Trivially, if the unconstrained optimal capital choice k^* is lower than λa, then profits don't depend on a. On the other hand, if the firm is constrained, meaning that $k^* > \lambda a$, then $k = \lambda a$, and a higher level of a results in strictly higher profits, as profits increase monotonically for values of $k < k^*$. Next, without loss of generality, we assume that firms are unconstrained to study how profits change with m and ν. Denote (k^*, l^*) the optimal capital and labor choices for (a, m, z) states and managerial skills are $m' > m$, then profits are

$$zm^{\epsilon}\left((k^*)^{\alpha}(l^*)^{1-\alpha}\right)^{\nu} - wl^* - rk^* - \phi(m) \leq z\left(m'\right)^{\epsilon}\left((k^*)^{\alpha}(l^*)^{1-\alpha}\right)^{\nu} - wl^* - rk^* - \phi\left(m'\right)$$
$$\leq \max_{k,l} zm^{\epsilon}\left(k^{\alpha}l^{1-\alpha}\right)^{\nu} - wl - rk - \phi\left(m'\right)$$

since $\epsilon \geq 0$ and $\phi'(m) \leq 0$, this shows that higher managerial skills imply greater profits. Analogously, if span-of-control $\nu' > \nu$ and Assumption 1 holds then

$$zm^{\epsilon}\left((k^*)^{\alpha}(l^*)^{1-\alpha}\right)^{\nu} - wl^* - rk^* \leq zm^{\epsilon}\left((k^*)^{\alpha}(l^*)^{1-\alpha}\right)^{\nu'} - wl^* - rk^*$$
$$\leq \max_{k,l} zm^{\epsilon}\left(k^{\alpha}l^{1-\alpha}\right)^{\nu'} - wl - rk,$$

which shows that a higher span of control implies higher profits. □

Lemma 1 shows that firms with greater owner's wealth (less constrained), better managers, or larger span-of-control will have higher profits, which could generate gains from trading firms. We impose an additional restriction to simplify the analysis of the span-of-control case in Proposition 1.[40]

Assumption 2. *We restrict the prices and parameters to be such that:*

$$zm^{\epsilon}\nu\left(\frac{\alpha}{R}\right)^{1-(1-\alpha)\nu}\left(\frac{(1-\alpha)}{w}\right)^{(1-\alpha)\nu} > 1.$$

[40]The restriction imposes that the initial span-of-control ν is in the region where capital is increasing with ν if unconstrained, to rule out that a larger span-of-control reduces capital.

Next, in our main proposition, we characterize how output, capital, ARPL, ARPK, profitability, and leverage change after a firm is traded due to (i) financial frictions, with the buyer having more assets a; (ii) managerial skills, with the buyer being a better manager m; and (iii) span-of-control, with the buyer's technology having a larger scale ν.

Proposition 1. *(Firms After Trade)*

Consider the problem of a firm that solves Equation (18).

(1) ***Financial Frictions.*** *If the new owner has more wealth a and the firm is constrained when sold, then the firm's output and capital increase, while ARPK, profitability, and leverage decrease, with ARPL remaining constant.*

(2) ***Managerial Skills.*** *If the new owner has a higher managerial skill m and the firm is unconstrained when sold output and capital will increase. Meanwhile, ARPK and ARPL will remain constant, and profitability and leverage will increase.*

(3) ***Span-of-Control.*** *Assuming that Assumption 1 and Assumption 2 hold, the new owner operates with a higher ν, and the firm is unconstrained when sold, then both the firm's output and capital increase. However, ARPL, ARPK, and profitability decrease while leverage increases.*

Proof. (Proposition 1) **Financial Frictions.** Consider the case where firms are constrained, then $k = \lambda a$ and output is $y = \left[\frac{zm^{\epsilon}\nu(1-\alpha)}{w}\right]^{\frac{1}{1-(1-\alpha)\nu}} (\lambda a)^{\frac{\alpha\nu}{1-(1-\alpha)\nu}}$, which are both increasing in a. Moreover, the ARPK, ARPL, and $\frac{\pi}{k}$ are

$$\frac{y}{k} = \left[\frac{zm^{\epsilon}\nu\left(1-\alpha\right)}{w}\right]^{\frac{1}{1-(1-\alpha)\nu}} (\lambda a)^{\frac{\nu-1}{1-(1-\alpha)\nu}}$$
$$\frac{y}{wl} = \frac{1}{\nu\left(1-\alpha\right)}$$
$$\frac{\pi}{k} = \left[\frac{zm^{\epsilon}\nu\left(1-\alpha\right)}{w}\right]^{\frac{1}{1-(1-\alpha)\nu}} (\lambda a)^{\frac{\nu-1}{1-(1-\alpha)\nu}} \left[1-\nu\left(1-\alpha\right)\right] - R$$

where ARPK and profitability are decreasing, and ARPL is constant in the owner's wealth a. For simplicity, we assumed $\phi\left(m\right) = 0$.[41] Finally, the leverage is $\frac{k-a}{k} = \frac{\lambda-1}{\lambda}$, which is unchanged as long as the increase in a is not enough to make the firm unconstrained. Thus, consider an increase in $a^{'} > a$ such that the firm under assets a is constrained, but

[41]It takes more assumptions to characterize the change in profitability if $\phi\left(m\right) > 0$.

under a' is unconstrained, then leverage for a' is

$$\frac{k-a'}{k}=1-\frac{a'}{k\left(z,m\right)^{*}}<\frac{\lambda-1}{\lambda}$$

where the unconstrained capital choice is $k\left(z,m\right)^{*}=\left[zm^{\epsilon}\nu\left(\frac{\alpha}{R}\right)^{1-(1-\alpha)\nu}\left(\frac{(1-\alpha)}{w}\right)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\nu}}$. Since $k\left(z,m\right)^{*}$ is not a function of a, leverage mechanically decreases with assets.

Managerial Skills. Consider the case where managerial ability m increases and $\phi(m)>0$. First, let's assume the firm is unconstrained. Then, capital and output choices are

$$k\left(z,m\right)^{*}=\left[zm^{\epsilon}\nu\left(\frac{\alpha}{R}\right)^{1-(1-\alpha)\nu}\left(\frac{(1-\alpha)}{w}\right)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\nu}}$$
$$y^{*}\left(z,m\right)=\left(zm^{\epsilon}\right)^{\frac{1}{1-\nu}}\left[\nu\left(\frac{1-\alpha}{w}\right)^{(1-\alpha)}\left(\frac{\alpha}{R}\right)^{\alpha}\right]^{\frac{\nu}{1-\nu}},$$

which are both strictly increasing in m for $\epsilon>0$. On the other hand, ARPK and ARPL are constant in managerial skills m and equal to

$$\frac{y}{lw}=\frac{1}{\nu\left(1-\alpha\right)}$$
$$\frac{y}{k}=\frac{R}{\nu\alpha}.$$

Finally, the firm's profitability is

$$\frac{\pi}{k}=\frac{R}{\nu\alpha}\left(1-\nu\left(1-\alpha\right)\right)-R-\frac{\phi\left(m\right)}{k},$$

since a larger m reduces $\phi(m)$ and increases k, then profitability increases when m increases. Finally, firms' leverage increases since a fixed and $k^{*}(z,m)$ increasing in m.[42]

Span-of-Control. Finally, we consider the case where span-of-control ν increases. Additionally, we assume that Assumption 1 and Assumption 2 hold, and for simplicity we assume $\phi\left(m\right)=0$. From the unconstrained output and capital equations derived above, an increase in ν results in higher output and capital. Furthermore, ARPL, ARPK, and profitability decrease with ν. Lastly, similar to the case of an increase in managerial skills, an increase in the span of control leads to higher leverage. □

Table C.1 summarizes the implications of Proposition 1 and compares them to the dynamics in the data for the six different variables of interest. The financial frictions motive is the only one consistent with the data for all these variables. In contrast, the managerial

[42] Dynamically, firms' leverage would also depend on owners' saving behavior. Under incomplete markets, the increase in income of the new owner may reduce the motives for saving due to lower precautionary behavior, increasing firms' leverage over time.

skills motive yields different implications for ARPK, profitability, and leverage compared to the data. Similarly, the span-of-control motive has different implications for ARPL and leverage relative to the post-trade firm dynamics in the data.

Table C.1: Post-Trade Dynamis in Proposition 1 and Data

	Variables					
	y	k	y/k	y/wl	π/k	b/k
Data	(+)	(+)	(−)	(=)	(−)	(−)
Theory						
Financial Frictions	(+)	(+)	(−)	(=)	(−)	(−)
Managerial Skills	(+)	(+)	(=)	(=)	(+)	(+)
Span-of-control	(+)	(+)	(−)	(−)	(−)	(+)

Notes: (+) implies that the variable increases after trade, (−) indicates a decrease, and (=) denotes no change. Debt is $b = k - a$. Theory results come from Proposition 1. Data refers to the empirical evidence documented in Section 2.2.

In addition to our main proposition, we extend Proposition 1 for the case when firms are constrained when sold. We show that when the firm is constrained, and financial frictions do not drive the trade, capital would not increase, which implies an increase in ARPK, and leverage would be constant for both managerial skills and span-of-control motives, which is at odds with the data. In the span-of-control case, it is unclear if profitability will decrease since the ARPK increases, but ARPL is decreasing.

Proposition 2. *(Firms After Trade when Constrained)*

Consider the problem of a firm that solves Equation (18) and is constrained when sold.

1. ***Managerial Skills.*** *If the new owner's managerial skills m are superior and the firm was constrained when sold, output increases, but capital is constant. Meanwhile, ARPK and profitability increase and ARPL and leverage remain constant.*

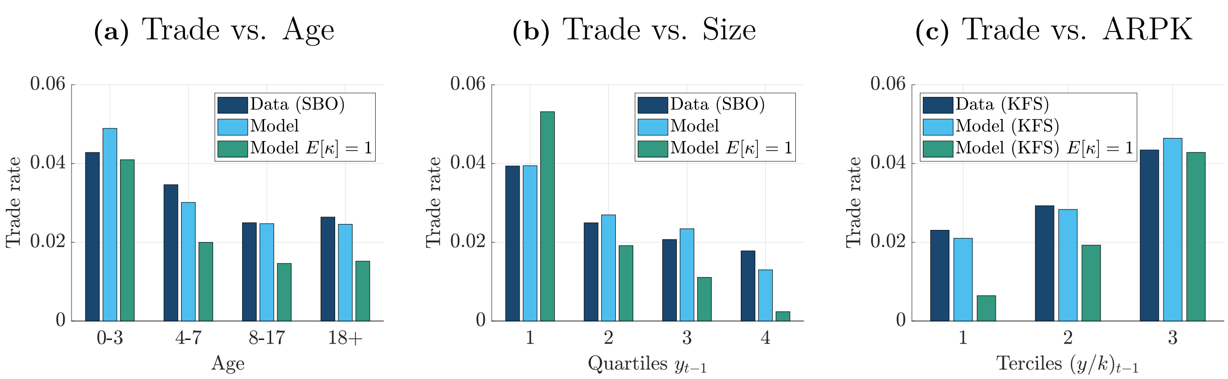
2. ***Span-of-Control.*** *Assuming that $\frac{zm^{\epsilon}\nu(1-\alpha)}{w}(\lambda a)^{\alpha\nu} > 1$ holds, if the new owner operates with a higher ν and the firm is constrained when sold, the output increases but capital is constant. Meanwhile, ARPL is lower, leverage is constant, ARPK increases, and profitability is ambiguous.*

Proof. (Proposition 2) **Managerial Skills.** Consider the case where managerial ability m increases and $\phi(m) > 0$. If the firm is constrained, $k = \lambda a$ is constant, and output increases with m (see proof of Proposition 1 for the financial frictions motive). Thus, the ARPK increases. ARPL, as before, remains constant since it is independent of financial frictions and production shifters (such as z and m). Leverage is trivially constant since a is fixed. Finally, according to the constrained ARPK equation in Proposition 1 for the financial frictions motive, profitability increases with m.

Span-of-Control. Consider the case where span-of-control ν increases. Additionally, we assume that $\frac{zm^{\epsilon}\nu(1-\alpha)}{w}(\lambda a)^{\alpha\nu} > 1$ holds. Again, capital is fixed since the firm is constrained and output increases. Thus, differently from the unconstrained case, ARPK increases, ARPL decreases (as in the unconstrained case). Still, profitability is ambiguous (see the profitability equation in Proposition 1 for the financial frictions motive). Intuitively, profitability increases due to greater ARPK but decreases due to lower ARPL, so it is undetermined. □

C.6 Additional Results

Figure C.1: Trade Rate by Firms' Characteristics with and without Preference Shocks



Notes: Trade rate by firms' characteristics in the data and data simulated from the model. To be consistent with the data, Model (KFS) restricts to a sample of firms of age less or equal to 7. See the notes in Figure 1 for a description of the data moments.

Table C.2: Untargeted Moments

	Data	Model		Data	Model
Income Distribution All Households			*Wealth Distribution All Households*		
Top 1	0.22	0.20	Top 1	0.33	0.40
Top 5	0.39	0.39	Top 5	0.60	0.62
Top 10	0.49	0.54	Top 10	0.72	0.75
Bottom 75	0.31	0.30	Bottom 75	0.13	0.07
Bottom 50	0.12	0.16	Bottom 50	0.02	0.01
Bottom 25	0.02	0.04	Bottom 25	0.00	0.00
Income Distribution Entrepreneurs			*Wealth Distribution Entrepreneurs*		
Top 1	0.23	0.36	Top 1	0.24	0.29
Top 5	0.44	0.67	Top 5	0.45	0.63
Top 10	0.57	0.81	Top 10	0.60	0.80
Bottom 75	0.24	0.15	Bottom 75	0.18	0.11
Bottom 50	0.10	0.11	Bottom 50	0.05	0.06
Bottom 25	0.03	0.07	Bottom 25	0.01	0.04

Source: 2007 SCF.

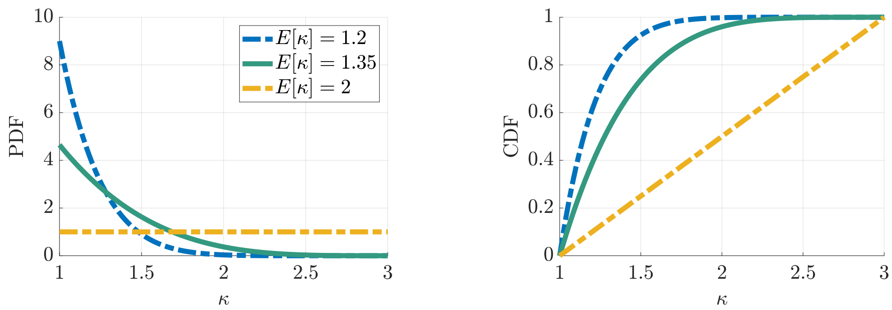
Table C.3: Wealth Ratio of Firm Buyers' to Households and Entrepreneurs

	Data	Model
Firm Buyers to Average Household		
Wealth $(a+p)$	3.83	3.09
Wealth Excluding Business Wealth (a)	2.71	2.74
Firm Buyers to Average Entrepreneur		
Wealth $(a+p)$	0.69	0.54
Wealth Excluding Business Wealth (a)	0.79	0.75

Source: 1989-2016 SCF.
Notes: We define firm buyers, in the SCF, as those entrepreneurs who purchased their primary business in the year of the survey or the previous one. We compute the ratio as the average wealth of firm buyers divided by the average wealth of all households or entrepreneurs. Because of the small sample of recent business buyers, we take the average across SCF waves. Entrepreneurs are defined as self-employed business owners who manage a business with at least one employee.

Figure C.2: Preference Shocks Distribution

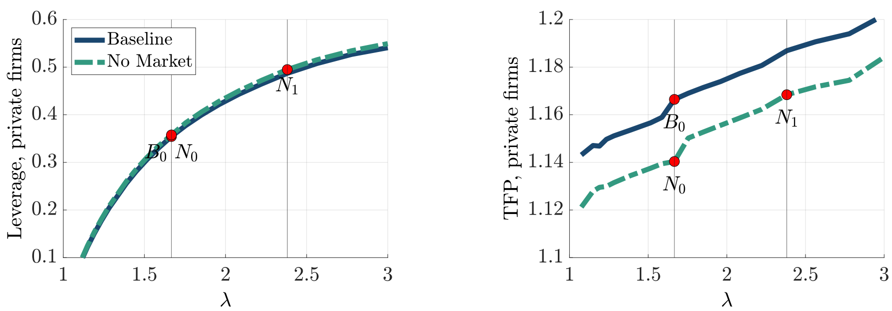
(a) PDF **(b)** CDF, $\Psi(\kappa)$



Notes: PDF and CDF of the preference shocks κ_{jt} defined in (6), considering the bounds $\underline{\kappa} = 1$ and $\overline{\kappa} = 3$. The panels present three distributions parameterized by preference shocks' mean $\mathbb{E}[\kappa]$. Given $\underline{\kappa}$ and $\overline{\kappa}$, this moment defines the parameters of the underlying Beta distribution of the auxiliary random variable ξ, as $\mathbb{E}[\kappa] = \underline{\kappa} + (\overline{\kappa} - \underline{\kappa})\mathbb{E}[\xi]$ and $\mathbb{E}[\xi] = \frac{1}{1+\beta_\kappa}$.

Figure C.3: Baseline vs. No Market Economy

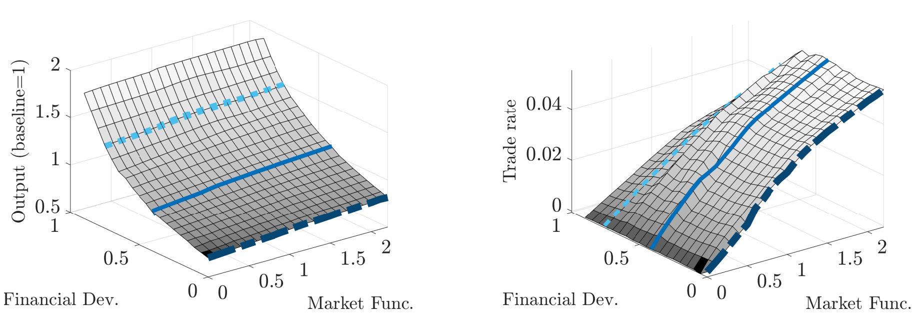
(a) Leverage **(b)** TFP



Notes: Steady-state values for the baseline and no market economy varying λ, which parameterizes firms' credit constraints. Panel (a) is private firms' mean leverage, $(k-a)/k$, weighted by capital k. Panel (b) is private firms' TFP. Points B_0 and N_0 denote the allocations in the baseline and no market economies. N_1 is the counterfactual no market economy that attains the same TFP as the baseline model.

Figure C.4: Financial Development and Functioning of the Market for Firms

(a) Private Firms' Output **(b)** Trade Rates



Notes: Financial Development is defined by firms' maximum leverage, $(\lambda - 1)/\lambda$. Market Functioning is parameterized by ω multiplying the search frictions in the market for firms $\alpha_o(\omega) = \min\{\omega\alpha_o\}, \alpha_w(\omega) = \min\{\omega\alpha_w, 1\}$.